Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Vor 2016

# Eðlisfræði II V

**Lokapróf**

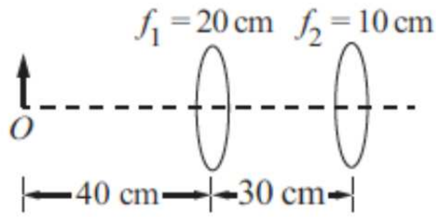
**6. maí 2016 kl. 13:30 - 16:30**

Leyfileg hjálpargögn eru skriffæri og vasareiknir

1. **Tveggja linsu kerfi – Two lens system** (9)

   Hlutur er staðsettur í 40 cm fjarlægð frá fyrri af tveimur þunnum safnlinsum sem hafa brennivíddir $f_1 = 20$ cm og $f_2 = 10$ cm eins og sýnt er á myndinni að neðan. Linsurnar eru aðskildar með 30 cm. Hvar kemur launmynd þessa tveggja linsu kerfis fram ? Er hún á hvolfi ?

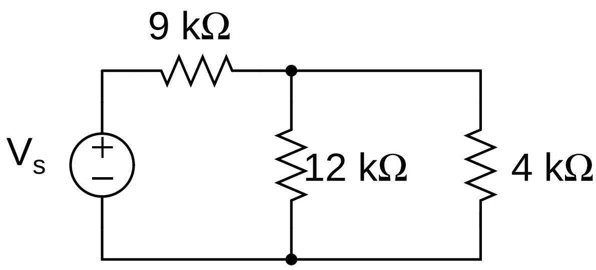
   An object is located 40 cm from the first of two thin converging lenses of focal lengths $f_1 = 20$ cm and $f_2 = 10$ cm as shown in the figure. The lenses are separated by 30 cm. Where is the final image of this two – lens system formed ? Is it inverted ?




2. **Spennulind og afl – Voltage source and power** (5)

   Gefið er að aflið sem eyðist í 4 kΩ viðnáminu er 144 mW. Finna skal $V_s$.

   The power that is dissipated in the 4 kΩ resistor is 144 mW. Find $V_s$.



3. **Gauss flötur – Gaussian surface** (9)

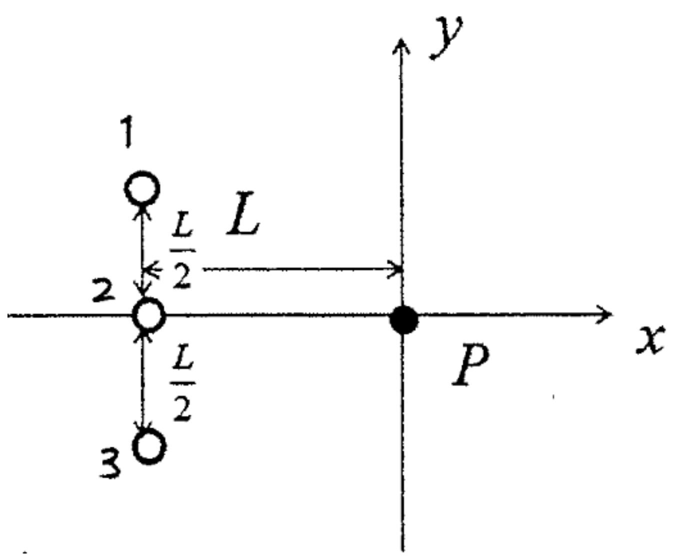
Hálfhvel af radía 3.5 cm inniheldur heildar hleðslu $6.6 \times 10^{-7}$ C. Flæðið um kúlulaga hluta yfirborðsins er $9.8 \times 10^4$ N m$^2$/C. Finna skal flæðið um flata grunninn.

A 3.5-cm radius hemisphere contains a total charge of $6.6 \times 10^{-7}$ C. The flux through the rounded portion of the surface is $9.8 \times 10^4$ N m$^2$/C. Find the flux through the flat base.

4. **Hleðsla og rafsvið – Charge and electric field** (14)

Neikvæðar hleðslur, hver af stærð $q$, eru staðsettar eins og sést á myndinni hér að neðan. Ákvarða skal jöfnu sem lýsir stærð og stefnu rafsviðsins í punktinum P (upphafspunkti).

Negative point charges, each of magnitude $q$, are located at positions shown in the figure below. Determine the expression for the magnitude and direction of the electric field at the point P (origin).



5. **LRC rás – LRC circuit** (11)

Drifin LRC rás samanstendur af raðtengingu á spólu $L = 1$ mH, viðnámi $R = 10\ \Omega$, og þéttis $C = 10\ \mu$F. Gerum ráð fyrir að hún sé drifin með íspennu $v(t) = V_\mathrm{m}\cos(\omega t)$. Ef $\omega_\mathrm{max}$ er horntíðnin þar sem straumútslagið er stærst, hvert er þá hlutfall straums við tvöfalda þá tíðni og hæsta straumgildis $I(2\omega_\mathrm{max})/I(\omega_\mathrm{max})$ ?

A driven LRC circuit consists of a series connection of an inductor $L = 1$ mH, a resistor $R = 10\ \Omega$, and a capacitor $C = 10\ \mu$F. Suppose it is driven by an EMF of $v(t) = V_\mathrm{m}\cos(\omega t)$. If $\omega_\mathrm{max}$ is the angular frequency at which the current amplitude is greatest, what is the ratio $I(2\omega_\mathrm{max})/I(\omega_\mathrm{max})$ of the current amplitude at twice this angular frequency to its maximum value ?

6. **Leifturljós – Flashlight** (12)

Leifturljós fær orku sína frá 150 $\mu$F þétti sem þarf 120 V spennu til að hann hleypi af. Ef þéttirinn er hlaðinn með 150 V rafhlöðu um 18 k$\Omega$ viðnám, hve lengi þarf ljósmyndarinn að bíða milli ljósblossa ? Gera skal ráð fyrir að þéttirinn sé full hlaðinn þegar af er hleypt.

A flashlight gets its energy from a 150 $\mu$F capacitor that requires 120 V voltage to operate. If the capacitor is charged with a 150 V battery through 18 k$\Omega$ resistor, how long does the photographer have to wait between flashes ? You can assume that the capacitor is fully charged when it is discharged.

7. **Rafeind í einsleitu rafsviði – Electron in a uniform electric field** (11)

Rafeind er send inn í einsleitt rafsvið $\mathbf{E} = 1000\ \hat{\mathbf{i}}$ N/C með upphafshraða $\mathbf{v}_0 = 2.00 \times 10^6\ \hat{\mathbf{i}}$ m/s í stefnu sviðsins. Hve langt ferðast rafeindin þangað til að hún stöðvast ?

An electron is projected into a uniform electric field $\mathbf{E} = 1000\ \hat{\mathbf{i}}$ N/C with an initial velocity $\mathbf{v}_0 = 2.00 \times 10^6\ \hat{\mathbf{i}}$ m/s in the direction of the field. How far does the electron travel before it is brought momentarily to rest ?

8. **Rafsvið við yfirborð hlaðins leiðara – electric field outside the surface of charged conductor** (3)

Í rafstöðufræði, rafsviðið rétt utan við yfirborð sérhvers hlaðins leiðara

(A) er alltaf samsíða yfirborðinu.

(B) er alltaf núll vegna þess að rafsviðið er núll innan leiðarans.

(C) er alltaf hornrétt á yfirborð leiðarans.

(D) er hornrétt á yfirborð leiðarans aðeins ef það er kúla, sívalningur eða flöt plata.

(E) getur haft ekki núll þætti hornrétt á og samsíða yfirborði leiðarans.

Under electrostatic conditions, the electric field just outside the surface of any charged conductor

(A) is always parallel to the surface.

(B) is always zero because the electric field is zero inside conductors.

(C) is always perpendicular to the surface of the conductor.

(D) is perpendicular to the surface of the conductor only if it is a sphere, a cylinder, or a flat sheet.

(E) can have nonzero components perpendicular to and parallel to the surface of the conductor.

9. **Raðtengd LRC rás – LRC series circuit** (3)

Þegar raðtengd LRC rás er í hermu, hver eftirfarandi staðhæfinga um rásina er rétt ? (Það kunna að vera fleiri en ein staðhæfing sem er rétt.)

(A) Samviðnámið er stærst.

(B) Launviðnám spólunnar er núll.

(C) Launviðnám þéttisins er núll.

(D) Launviðnám vegna spólu og þéttis hafa stærsta gildi sitt.

(E) Straumútslagið er í sínu stærsta gildi.

When an LRC series circuit is at resonance, which one of the following statements about that circuit is accurate? (There may be more than one correct choice.)

(A) The impedance has its maximum value.

(B) The reactance of the inductor is zero.

(C) The reactance of the capacitor is zero.

(D) The reactance due to the inductor and capacitor has its maximum value.

(E) The current amplitude is a maximum.

10. **Segulflæði – Magnetic flux** (17)

Hér að neðan sjáum við leiðandi fernings lykkju. Lykkjan er ekki hreyfanleg. Hliðarlengdir lykkjunnar eru 1 m. Hægri helmingur lykkjunnar er innan einsleits segulsviðs (skyggða svæðið), sem stefnir út úr plani pappírsins. Viðnám lykkjunnar er 1 Ω.

(a) Við $t = 0$, er styrkur segulsviðsins $B = 2$ T. Hver er styrkur segulflæðisins um lykkjuna á þessu augnabliki ?

(b) Frá tímanum $t = 1$ s, er segulsviðið lækkað frá $B = 2$ T til $B = 0$ yfir 1 s tímabil eð með föstum hraða. Hver er stærð spanaðrar íspennu við $t = 1.5$ s. Sýnið útreikninga.

(c) Hver er stefna og stærð spanaða straumsins við $t = 1.5$ s ?

(d) Hver er stefna of stærð heildar segulkraftsins sem verkar á lykkjuna við $t = 1.5$ s ?

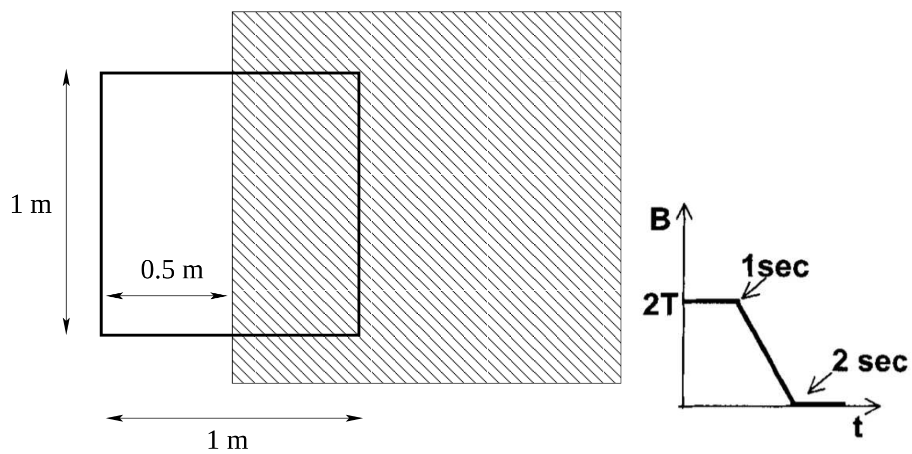
Shown below is a square conducting loop. The loop is not movable. The sides of the loop have length 1 m. The right half of the loop is inside a uniform external magnetic field (the shaded area), which points out of the paper plane. The reisistance of the loop is 1 Ω.

(a) At time $t = 0$, the magnitude of the field is $B = 2$ T. What is the magnitude of the magnetic flux through the loop at this time ?

(b) Starting at time $t = 1$ s, the field is ramped from $B = 2$ T to $B = 0$ over the course of 1 s with a constant rate. What is the magnitude of the induced emf at $t = 1.5$ s during the ramp. Show work.

(c) What is the direction and magnitude of the induced current at $t = 1.5$ s ?

(d) What is the direction and magnitude of the net magnetic force on the loop at $t = 1.5$ s ?



11. **Raðtengd LRC rás – LRC series circuit** (3)

Í raðtengdri LRC rás, er hermitíðnin $f_0$. Ef að nú viðnámið, spanið, rýmdin og spennuútslagið eru tvöfölduð, hver er ný hermitíðni ?

(A) $4f_0$

(B) $2f_0$

(C) $f_0$

(D) $f_0/2$

(E) $f_0/4$

In a series LRC circuit, the frequency at which the circuit is at resonance is $f_0$. If you double the resistance, the inductance, the capacitance, and the voltage amplitude of the ac source, what is the new resonance frequency ?

(A) $4f_0$

(B) $2f_0$

(C) $f_0$

(D) $f_0/2$

(E) $f_0/4$

12. **Virk spenna – rms voltage** (3)

Riðstraum er veitt um tól sem hefur á sér þá viðvörun að spennan yfir tólið sé aldrei hærri en 12 V. Hver er hæsta rms spennan sem leggja má yfir tól þetta en þó þannig að við séum innan mörkin ?

(A) $6\sqrt{2}$ V

(B) $12\sqrt{2}$ V

(C) 144 V

(D) 6 V

An alternating current is supplied to an electronic component with a warning that the voltage across it should never exceed 12 V. What is the highest rms voltage that can be supplied to this component while staying below the voltage limit in the warning?

(A) $6\sqrt{2}$ V

(B) $12\sqrt{2}$ V

(C) 144 V

(D) 6 V

# J Ö F N U R

## RAFSTÖÐUFRÆÐI

$k = 1/(4\pi\epsilon_0)$

Coulomb $\mathbf{F} = \frac{kqQ}{r^2}\hat{\mathbf{r}}$

Rafsvið

$$\mathbf{E} = \frac{kQ}{r^2}\hat{\mathbf{r}} \quad \mathbf{E} = k\int \frac{dq}{r^2}\hat{\mathbf{r}}$$

Umhverfis langan vír $E = 2k\lambda/R$

Við þynnu $E = \sigma/2\epsilon_0$

Milli þynna, þéttir $E = \sigma/\epsilon_0$

Tvískautsvægi $\mathbf{p} = q\,\mathbf{d}$

$\mathbf{E} = k(-\mathbf{p}/r^3 + 3(\mathbf{p}\cdot\mathbf{r})\mathbf{r}/r^5)$

$\boldsymbol{\tau} = \mathbf{p}\times\mathbf{E}$

$U = -\mathbf{p}\cdot\mathbf{E}$

$\mathbf{F} = \nabla(\mathbf{p}\cdot\mathbf{E})$

Raffllæði $\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A}$

Lögmál Gauss $\oint \mathbf{E}\cdot d\mathbf{A} = \frac{Q}{\epsilon_0}$

Rafmætti

$$V_B - V_A = -\int_A^B \mathbf{E}\cdot d\mathbf{s}$$

í einsleitu sviði $\Delta V = \pm E\,d$

fyrir punkthleðslu $V = \frac{kQ}{r}$

dreifð hleðsla $V = \int \frac{k\,dq}{r}$

Mættisorka $U = q\,V$

$\mathbf{E} = -\nabla V$

Rýmd $C = Q/V$

Plötuþéttir $C = \epsilon_0 A/d$

Hliðt. $C_{eq} = C_1 + C_2 + \ldots + C_N$

Raðt. $1/C_{eq} = 1/C_1 + 1/C_2 + \ldots + 1/C_N$

$U_E = Q^2/2C = QV/2 = CV^2/2$

Orkuþéttleiki $u_E = \frac{1}{2}\,\epsilon_0 E^2$

Rafsvari $C = \kappa C_0 \quad E_D = E_0/\kappa$

## RAFSEGULFRÆÐI

Straumur $I = dQ/dt \quad J = I/A$

$\mathbf{J} = nq\,\mathbf{v}_d \quad \mathbf{J} = (1/\rho)\mathbf{E} = \sigma\mathbf{E}$

$R = V/I \quad R = \rho l/A \quad V = IR$

$P = IV = I^2R = V^2/R$

$\rho = \rho_0(1 + \alpha(T - T_0))$

Kirchhoff: $\boldsymbol{\Sigma}I = 0 \quad \boldsymbol{\Sigma}V = 0$

Viðnám

raðt. $R_{eq} = R_1 + R_2 + \ldots + R_N$

hliðt. $1/R_{eq} = 1/R_1 + 1/R_2 + \ldots + 1/R_N$

Afhleðsla og hleðsla þéttis

$Q = Q_0 e^{-t/\tau}$; $I = I_0 e^{-t/\tau}$

$\tau = RC$

$Q = Q_0(1 - e^{-t/\tau})$; $I = I_0 e^{-t/\tau}$

$\mathbf{F} = q\,\mathbf{v}\times\mathbf{B} \qquad \mathbf{F} = I\boldsymbol{\ell}\times\mathbf{B}$

$d\mathbf{F} = I\,d\mathbf{l}\times\mathbf{B} \qquad \boldsymbol{\mu} = NIA\,\hat{\mathbf{n}}$

$\boldsymbol{\tau} = \boldsymbol{\mu}\times\mathbf{B} \qquad U = -\boldsymbol{\mu}\cdot\mathbf{B}$

Rafeind á braut $evB = mv^2/r$

$\boldsymbol{\mu} = -(e/2m)\mathbf{L} \quad L = mvr$

Lorentz $\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B})$

Hall $nq = \frac{-J_x B_y}{E_z}$

Langur vír $B = \mu_0 I/2\pi R$

Tveir vírar $F/\ell = \frac{\mu_0 I_1 I_2}{2\pi d}$

Biot-Savart

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\boldsymbol{\ell}\times\hat{\mathbf{r}}}{r^2}$$

Spóla $B = \frac{1}{2}\mu_0 nI(\sin\theta_2 - \sin\theta_1)$

Ampere $\oint \mathbf{B}\cdot d\boldsymbol{\ell} = \mu_0 I$

Segulflæði $\Phi_B = \mathbf{B}\cdot\mathbf{A}$

$\Phi_B = \int \mathbf{B}\cdot d\mathbf{A}$

Faraday $\mathcal{E} = -\frac{d\Phi_B}{dt}$

Sjálfspan, víxlspan

$\mathcal{E}_{11} = -L_1\frac{dI_1}{dt}$ $\quad N_1\Phi_{11} = L_1 I_1$

$\mathcal{E}_{12} = -M\frac{dI_2}{dt}$ $\quad N_1\Phi_{12} = MI_2$

Spóla $L = \mu_0 n^2 A\ell$

LR-rás

$I = I_0(1 - e^{-t/\tau})$

$I = I_0 e^{-t/\tau}$

$\tau = \frac{L}{R}$ $\quad I_0 = \mathcal{E}/R$

Orka í spólu $U_L = \frac{1}{2}LI^2$

Orkuþéttleiki $u_B = \frac{B^2}{2\mu_0}$

LC- sveiflur

$$\frac{d^2Q}{dt^2} + \frac{1}{LC}Q = 0 \quad \omega_0 = \frac{1}{\sqrt{LC}}$$

Deyfðar LC-sveiflur

$$L\frac{d^2Q}{dt^2} + R\frac{dQ}{dt} + \frac{Q}{C} = 0$$

$$\begin{aligned} Q &= Q_0 e^{-Rt/2L}\cos(\omega' t + \delta) \\ \omega' &= \sqrt{\omega_0^2 - \left(\frac{R}{2L}\right)^2} \end{aligned}$$

Riðstraumur $i = i_0\sin(\omega t)$

$v = v_0\sin(\omega t + \phi)$

rms gildi

$I = \sqrt{(i^2)_{av}} = \frac{i_0}{\sqrt{2}} \approx 0.707 i_0$

$V = \sqrt{(v^2)_{av}} = \frac{v_0}{\sqrt{2}} \approx 0.707 v_0$

$X_L = \omega L \quad X_C = 1/\omega C$

Samviðnám $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\tan\phi = \frac{X_L - X_C}{R}$

Afl

$$\begin{aligned} P &= I^2 R = IV\cos\phi \\ P &= \frac{V^2 R}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \end{aligned}$$

Spennir $i_2 N_2 = i_1 N_1 \quad i_2 v_2 = i_1 v_1$

Maxwell

$$\begin{aligned} \oint \mathbf{E}\cdot d\mathbf{A} &= \frac{Q}{\epsilon_0} \\ \oint \mathbf{B}\cdot d\mathbf{A} &= 0 \\ \oint \mathbf{E}\cdot d\boldsymbol{\ell} &= -\frac{d\Phi_B}{dt} \\ \oint \mathbf{B}\cdot d\boldsymbol{\ell} &= \mu_0\left(I + \epsilon_0\frac{d\Phi_E}{dt}\right) \end{aligned}$$

Á diffurformi

$$\begin{aligned} \nabla\cdot\mathbf{E} &= \frac{\rho}{\epsilon_0} \\ \nabla\cdot\mathbf{B} &= 0 \\ \nabla\times\mathbf{E} &= -\frac{\partial\mathbf{B}}{\partial t} \\ \nabla\times\mathbf{B} &= \mu_0\left(\mathbf{J} + \epsilon_0\frac{\partial\mathbf{E}}{\partial t}\right) \end{aligned}$$

Öldulíkingar

$$\begin{aligned} \nabla^2\mathbf{E} - \mu_0\epsilon_0\frac{\partial^2\mathbf{E}}{\partial t^2} &= 0 \\ \nabla^2\mathbf{B} - \mu_0\epsilon_0\frac{\partial^2\mathbf{B}}{\partial t^2} &= 0\,. \end{aligned}$$

Um þætti sléttrar bylgju

$$\mathbf{E} = E_0\sin(kz - \omega t)\hat{\mathbf{x}}$$

$$\mathbf{B} = \frac{1}{c}\hat{\mathbf{k}}\times\mathbf{E}$$

$$\mathbf{E} = -c\hat{\mathbf{k}}\times\mathbf{B}$$

$c = (\mu_0\epsilon_0)^{-1/2} \quad E = cB \quad c = \lambda f$

Orkuþéttleiki

$$u = \epsilon_0 E^2 = \frac{B^2}{\mu_0} = \sqrt{\frac{\epsilon_0}{\mu_0}}EB$$

Poynting-vigur

$$\mathbf{S} = \frac{\mathbf{E}\times\mathbf{B}}{\mu_0} \quad S_{ave} = \frac{E_0 B_0}{2\mu_0}$$

Skriðþungi, geislaþrýstingur

$$p = \frac{U}{c} \quad \frac{F}{A} = \frac{S}{c} = u$$

## LJÓSFRÆÐI

$n = \frac{c}{v}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}$

$m = \frac{y_1}{y_0} = -\frac{q}{p}$

$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$

$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$

$\delta = m\lambda$

$\quad m = 0, \pm 1, \pm 2, \ldots$

$\delta = \left(m + \frac{1}{2}\right)\lambda$

$d \sin\theta = m\lambda$

$d \sin\theta = \left(m + \frac{1}{2}\right)\lambda$

$I = 4I_0 \cos^2(\frac{\phi}{2})$

$a \sin\theta = m\lambda$

$d \sin\theta = m\lambda$

$I = I_0 \frac{\sin^2(\alpha/2)}{(\alpha/2)^2} \quad \alpha = 2\pi a \sin\theta/\lambda$

Skautun

Malus $\quad I = I_{max}\cos^2\phi$

Brewster $\quad \tan\theta_p = \frac{n_b}{n_a}$

## INNGANGUR AÐ SKAMMTAFRÆÐI

Óvissulögmál Heisenberg

$\Delta x \Delta p \geq h$

$\Delta t \Delta E \geq \hbar/2$

## ATÓM OG KJARNEÐLISFRÆÐI

$\lambda = \frac{h}{p} = \frac{h}{mv}$

$hf = \frac{hc}{\lambda} = E_\mathrm{f} - E_\mathrm{i}$

Vetnislíkan Bohr

$E_n = K_n + U_n = -\frac{1}{\epsilon_0^2}\frac{me^4}{8n^2h^2} = -\frac{hcR}{n^2}$

$L_n = mv_n r_n = n\frac{h}{2\pi}$

Svarthlutageislunarlíkan Rayleigh

$I(\lambda) = \frac{2\pi ckT}{\lambda^4}$

Svarthlutageislunarlíkan Planck

$I(\lambda) = \frac{2\pi hc^2}{\lambda^5(\exp(hc/\lambda kT)-1)}$

1 u = 1 amu = 1 Da = 1.660538921 $\times 10^{-27}$ kg

1 u = 931.494095 MeV/$c^2$

$m_\mathrm{e} = 0.511$ MeV/$c^2$

$E_\mathrm{rest} = m_0 c^2$

$E_\mathrm{total} = mc^2$

$\alpha(t) = \lambda n(t)$

$T_{1/2} = \frac{\ln 2}{\lambda}$

Massajafna Bethe og Weizsäker

$\mathrm{BE} = a_\mathrm{v} A - a_\mathrm{s} A^{2/3} - a_\mathrm{c}\frac{Z^2}{A^{1/3}} - a_\mathrm{a}\frac{(N-Z)^2}{A} + \delta(A)$

## STÆRÐFRÆÐI

$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$

Vigrar:

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = A_x B_x + A_y B_y + A_z B_z$

$$\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$|\mathbf{A}\times\mathbf{B}| = AB\sin\theta$

$\hat{\mathbf{C}} = \frac{\mathbf{C}}{|\mathbf{C}|}$

Hornafræði:

$C^2 = A^2 + B^2 - 2AB\cos\gamma$

$\frac{\sin\alpha}{A} = \frac{\sin\beta}{B} = \frac{\sin\gamma}{C}$

$\sin(x+y) = \sin x\cos y + \cos x\sin y$

$\cos(x+y) = \cos x\cos y - \sin x\sin y$

$\sin x + \sin y = 2\sin(\frac{x+y}{2})\cos(\frac{x-y}{2})$

$\cos x + \cos y = 2\cos(\frac{x+y}{2})\cos(\frac{x-y}{2})$

$\sin(-x) = -\sin x$

$\cos(-x) = \cos x$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = 2\cos^2 x - 1$

Raðir:

$(1+x)^n = 1 + xn + \frac{n(n-1)}{2}x^2 + o(x^3)$

$\ln(1+x) = x - \frac{1}{2}x^2 + o(x^3)$

$\sin x = x + o(x^3)$

$\cos x = 1 - x^2/2 + o(x^4)$

$e^x = 1 + x + o(x^2)$

Rúmheildi:

Kartísk hnit

$$\int dx \int dy \int dz\ f(x,y,z)$$

Sívalnings hnit

$$\int_0^R \rho d\rho \int_0^{2\pi} d\varphi \int_0^h dz\ f(\rho,\varphi,z)$$

Kúluhnit

$$\int_0^R r^2 dr \int_0^{2\pi} d\varphi \int_0^\pi \sin\theta d\theta\ f(r,\varphi,\theta)$$

Diffurvirkjar:

Kartísk hnit

$$\nabla V = \hat{\mathbf{x}}\,\frac{\partial V}{\partial x} + \hat{\mathbf{y}}\,\frac{\partial V}{\partial y} + \hat{\mathbf{z}}\,\frac{\partial V}{\partial z}$$

$$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$$

Sívalningshnit

$$\nabla V = \hat{\rho}\,\frac{\partial V}{\partial \rho} + \hat{\phi}\,\frac{1}{\rho}\frac{\partial V}{\partial \phi} + \hat{\mathbf{z}}\,\frac{\partial V}{\partial z}$$

$$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$$

Kúluhnit

$$\nabla V = \hat{\mathbf{r}}\,\frac{\partial V}{\partial r} + \hat{\theta}\,\frac{1}{r}\frac{\partial V}{\partial \theta} + \hat{\phi}\,\frac{1}{r\sin\theta}\frac{\partial V}{\partial \phi}$$

$$\begin{aligned}\nabla^2 V \quad &= \quad \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) \\ &+ \quad \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) \\ &+ \quad \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}\end{aligned}$$