Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2015

Eðlisfræði þéttefnis I

Dæmablað 10

Skilafrestur 10. November 2015 kl. 15:00

1. **Drude model frequency dependence** (10)

   Use the equation
   $$m\left(\frac{dv_{\mathrm{D}}}{dt}+\frac{v_{\mathrm{D}}}{\tau}\right)=-e\mathcal{E}$$
   for the electron drift velocity v_{D} to show that the conductivity at frequency ω is
   $$\sigma(\omega)=\sigma(0)\left(\frac{1+j\omega\tau}{1+(\omega\tau)^2}\right)$$
   where $\sigma(0)=ne^2\tau/m$.

   (Próf Desember 2014)

2. **Conduction electrons in a metal with a uniform static electric field** (20)

   A uniform static electric field E is established in a metal at uniform temperature with a conduction electron number density n and relaxation time τ. An electron is considered to undergo scattering at time $t=0$ and then again at time t. Answer the following questions applying Drude's theory of electrons in metals.

   (a) What is the average energy lost by the electron in the second collision ?

   (b) What is the average energy loss of the electron per collision ?

   (c) Consider a sample of cross-section area A and length L along which a current I is flowing. Using your above result, demonstrate the total power dissipated is $P=RI^2$ and provide an expression for R.

(d) Suppose now that the metal has a uniform temperature gradient ∇T. The average energy of an electron at temperature T is $\mathcal{E}(T)$. Show that the temperature gradient is responsible for an extra term proportional to $\nabla T \cdot E$ in the average energy loss of an electron per collision and provide the expression of the proportionality constant.

(e) Consider again a sample of cross-section area A and length L with an electric potential difference ΔV and a temperature difference ΔT maintained between the two ends. Assuming the electric potential and temperature gradients are parallel and uniform, find a relation between ΔV and ΔT so the average energy loss per scattering cancels. (This is the main idea behind the Seebeck effect observed for the first time in 1821: in a metal subject to a temperature gradient, an electric field establishes itself in a direction opposite to the temperature gradient. This behavior is exploited in thermocouples used for temperature difference measurements.)

3. **The Kronig-Penney model** (20)

Consider an electron in 1D in the presence of the periodic potential (Kronig-Penney model)

$$U(x) = \sum_{m=-\infty}^{\infty} U_0 \Theta(x - ma)\Theta(ma + b - x)$$

(a) Restrict your attention to a single unit cell, and write down the boundary conditions for the wave function as required by Bloch's theorem.

(b) Solve the Schrödinger equation by constructing ψ(x) from plane waves and imposing suitable boundary conditions at $x = 0, b, a$. The results is a relation between the Bloch index k and the energy.

(c) Take the limit $b \longrightarrow 0$, $U_0 \longrightarrow \infty$ with $U_0 b \longrightarrow W_0 \frac{\hbar^2 a^{-2}}{m}$. Show that the condition for the Bloch index simplifies to

$$\cos(ka) = \frac{W_0}{qa}\sin(qa) + \cos(qa)$$

where q is related to the eigenenergy $\mathcal{E}$ via $q = (2m\mathcal{E}/\hbar^2)^{1/2}$.

(d) Produce plots of the lowest two energy bands $\mathcal{E}_{nk}(n = 0, 1)$ in the limit of part (c) with $a = 1$, $m = 1$, $\hbar = 1$, and $W_0 = 0.5$.