Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2015

Eðlisfræði þéttefnis I

Dæmablað 6

Skilafrestur 6. October 2015 kl. 15:00

1. **Atomic X-ray form factor** (15)

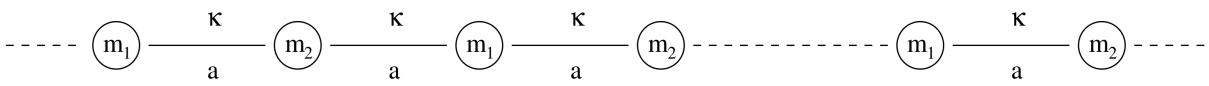
   As a first step toward including the atomic form factor for X-ray diffraction, one can consider the Z electrons of an atom to be uniformly distributed in a sphere of radius r_0. Show that the form factor of a uniform sphere of radius r_0 can be written

   $$f(\mathbf{G}) = 3Z\frac{\sin x_0 - x_0 \cos x_0}{x_0^3}$$

   with $x_0 = |\mathbf{G}r_0|$.

2. **Normal modes of a one dimensional diatomic crystal** (20)

   Consider a straight chain of atoms with alternating mass m_1 and m_2 and interatomic distance a. Nearest neighbors interact through a spring of constant κ.



   (a) Estalish the dispersion relation for the normal modes of the chain.

   (b) Discuss the cases $m_1 = m_2$ and $m_1 \gg m_2$ making use of reduced and extended zone representations.

3. **Dispersion relation** (15)

Consider 1D chain with identical masses M. Assume that there are nearest neighbor(nn) springs with spring constant K_1 and next nearest neighbor(nnn) springs with spring constant $K_2 < K_1$.

(a) Find the dispersion relation for this system.

(b) Calculate the speed of sound and compare to a system without nnn interactions.

(c) Now generalize to a new energy:

$$U^{\text{harmonic}} = \sum_n \sum_{m>0} K_m \left[u(na) - u([n+m]\,a)\right]^2$$

(i) Show that the dispersion relation is:

$$\omega = 2\sqrt{\sum_{m>0} K_m \frac{\sin^2(\frac{1}{2}mka)}{M}}$$

(ii) Show that the long wavelength limit of the dispersion relation for $K_m = K_1/m^p$ when $p = 3$ is

$$\omega \sim k\sqrt{|\ln k|}$$