Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2015

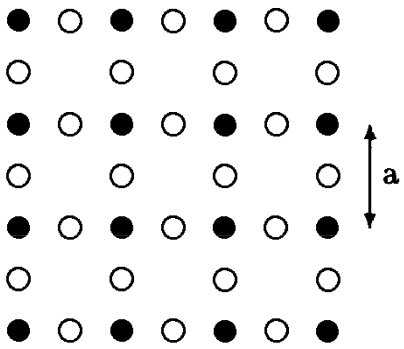
Eðlisfræði þéttefnis I

Dæmablað 7

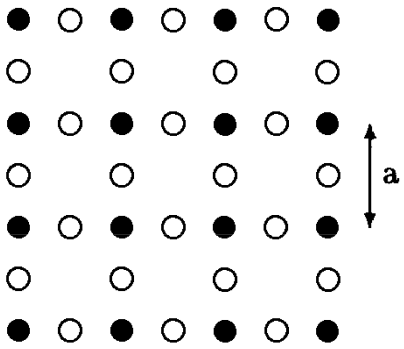
Skilafrestur 20. October 2014 kl. 15:00

1. **Copper Oxide Layers** (15)

   The common building blocks for most high temperature (high T_c) superconductors are copper oxide layers, as shown below. Assume the distance between copper atoms (filled circles) is a. For simplicity let us also assume that in the third dimension these CuO_2 layers are simply stacked with spacing c, and there are no other atoms in the crystal. In first approximation the layers have a four-fold symmetry; the crystal is tetragonal.



   (a) Sketch the Bravais lattice and indicate a possible set of primitive vectors for this crystal. What is the unit cell, and what is the basis ?



(b) In $LaCuO_4$ one discovers, at closer inspection, that the CuO_2 lattice is actually not flat, but that the oxygen atoms are moved a small amount out of the plane ("up" or "down") in an alternating fashion (in the figure a + meand up and a - means down). What is the primitive cell and lattice spacing for this crystal ? What is the reciprocal lattice ? Describe (qualitatively) what happens in the X-ray diffraction pattern as the distortion is decreased gradually to zero.

$LaCuO_4$ is an antiferromagnetic insulator. High temperature superconductivity was discovered in a closely related compound $La_{1-x}Ba_xCuO_4$. See J. G. Bednorz and K. A. Müller, *Z. Physik B* **64**, 189 (1986).

2. **Low-temperature specific heat in d dimensions and for nonlinear dispersion** (15)

Consider small lattice vibrations in a d-dimensional crystal in harmonic approximation.

(a) For the Debye model, i.e. a linear dispersion $\omega = c\,|k|$ of all phonon modes, calculate the phonon density of states and show that it varies as ω^{d-1}. What is the Debye frequency ?

(b) Determine the phonon contribution to low-temperature specific heat.

(c) Investigate what would happen for a nonlinear phonon dispersion $\omega \sim |k|^{\nu}$ (anomalous sound). Show that the low-temperature specific heat would vanish as $T^{d/\nu}$ in d dimensions.