Eðlisfræði þéttefnis I

Dæmablað 8

Skilafrestur 27. October 2015 kl. 15:00

1. Electron gas in two dimensions (20)

We consider a two dimensional gas of electrons of mass m with N electrons confined to an area A so there is a density n = N/A of electrons per unit area.

(a) Express the Fermi wave vector magnitude $k_{\rm F}$ and the Fermi energy $\mathcal{E}_{\rm F}$ in terms of n

(b) Express the density of levels $g(\mathcal{E})$

(c) Write the Sommerfeld expansion for n and conclude as to the relation between the chemical potential μ and the Fermi energy $\mathcal{E}_{\rm F}$

(d) Obtain a relation between μ and $\mathcal{E}_{\rm F}$ directly from the relation

$$n = \int_{-\infty}^{\infty} \mathrm{d}\mathcal{E}g(\mathcal{E})f(\mathcal{E})$$

where $f(\mathcal{E})$ is the Fermi-Dirac occupation factor (Hint: proceed with the change of variable $x = e^{-(\mathcal{E}-\mu)/k_{\rm B}T}$). Write your result to the limit $k_{\rm B}T \ll \mathcal{E}_{\rm F}$.

(e) Comment on the difference between your answers to question (c) and (d).

2. Fermi temperature (10)

Calculate the Fermi temperatures $T_{\rm F}$ for Cu, Na, and Ag. Also calculate the ratio $T/T_{\rm F}$ in each case for T = 300 K. The effective mass of Cu and Na are 1.0 and 1.2 times m_0 .

3. Einstein and quantum theory (10)

Describe how Einstein used quantum theory to explain the low-temperature behavior of the specific heat in solids. Use more than four sentences in your response.