Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2016

Eðlisfræði þéttefnis I

Dæmablað 10

Skilafrestur 22. November 2016 kl. 15:00

1. **Conduction mechanism of a solid** (15)

Myndin hér að neðan sýnir eðlisviðnám þéttefnis sem fall af hitastigi ($ErRhB_4$ óhreint, en ekki viljandi skemmt).

(a) Útfrá grafinu segið hvort þéttefnið er málmur eða einangrari.

(b) Lýsið megin ferlunum sem stýra leininni á eftirfarandi hitastigsbilum:

(1) T mjög nærri 0 K

(2) T nærri 25 K

(3) T nærri 300 K

(c) Áætlið meðal frjálsa vegalengd og meðal frjálsan tíma við $T = 0$ K og $T = 300$ K. Er þetta góður málmur ?

Nota má eftrifarandi gildi: $n = 10^{23}$ cm^{-3} og $v_\mathrm{F} = 10^8$ cm/s.

The figure below shows a rough plot of the electrical resistance of a solid ($ErRhB_4$ impure, but not purposely damaged).

(a) From the graph, is this material a metal or an insulator ?

(b) Describe the principal physical processes that account for the following three temperature regions:

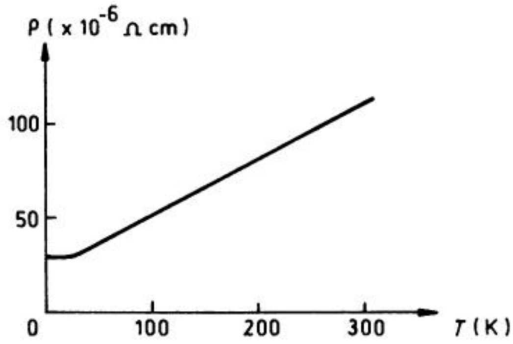
(1) T very near 0 K

(2) T near 25 K

(3) T near 300 K

(c) Estimate the mean free path and mean free time at $T = 0$ K and $T = 300$ K. Is the material a good metal ?

Useful parameters: $n = 10^{23}$ cm^{-3} og $v_{\mathrm{F}} = 10^{8}$ cm/s.



(Próf desember 2015)

2. **Conductivity of a metal** (10)

Notið jöfnuna

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = -e\mathbf{E}$$

fyrir skriðþunga rafeinda til að finna AC leiðni málms. Svar þitt skal vera á forminu $\sigma(\omega, n, e, \tau)$.

Use the equation

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = -e\mathbf{E}$$

for the electron momentum to find the AC conductivity of a metal. Your answer should be of the form $\sigma(\omega, n, e, \tau)$.

(Próf desember 2015)

3. **The Kronig-Penney model** (20)

Consider an electron in 1D in the presence of the periodic potential (Kronig-Penney model)

$$U(x) = \sum_{m=-\infty}^{\infty} U_0 \Theta(x - ma)\Theta(ma + b - x)$$

(a) Restrict your attention to a single unit cell, and write down the boundary conditions for the wave function as required by Bloch's theorem.

(b) Solve the Schrödinger equation by constructing ψ(x) from plane waves and imposing suitable boundary conditions at $x = 0, b, a$. The results is a relation between the Bloch index k and the energy.

(c) Take the limit $b \longrightarrow 0$, $U_0 \longrightarrow \infty$ with $U_0 b \longrightarrow W_0 \frac{\hbar^2 a^{-2}}{m}$. Show that the condition for the Bloch index simplifies to

$$\cos(ka) = \frac{W_0}{qa}\sin(qa) + \cos(qa)$$

where q is related to the eigenenergy $\mathcal{E}$ via $q = (2m\mathcal{E}/\hbar^2)^{1/2}$.

(d) Produce plots of the lowest two energy bands $\mathcal{E}_{nk}(n = 0, 1)$ in the limit of part (c) with $a = 1$, $m = 1$, $\hbar = 1$, and $W_0 = 0.5$.

4. **Fermi level adjustment in silicon** (10)

Kísilsýni við 300 K er íbætt með rafþega íbót af þéttleika $N_\mathrm{A} = 5 \times 10^{16}$ cm^{-3}. Ákvarða íbótarþéttleika rafgjafa íbótar sem bæta verður við þannig að kísillinn verði $n-$leiðandi og Fermi orkustigið sé 0.12 eV neðan við leiðniborðabrún. Virkur ástandsþéttleiki leiðniborða kísils er $N_\mathrm{C} = 2.86 \times 10^{19}$ cm^{-3}.

A silicon sample at 300 K contains an acceptor impurity concentartion of $N_\mathrm{A} = 5 \times 10^{16}$ cm^{-3}. Determine the concentration of donor impurity atoms that must be added so that the silicon is $n-$type and the Fermi level is 0.12 eV below the conduction band edge.

(Próf maí 2016)