

Eðlisfræði þéttfnis I

Dæmablað 7

Skilafrestur 25. October 2016 kl. 15:00

1. Dispersion relation (15)

Consider a linear chain in which alternate ions have mass M_1 and M_2 , and only nearest neighbors interact.

(a) Show that the dispersion relation for normal modes is

$$\omega^2 = \frac{K}{M_1 M_2} \left(M_1 + M_2 \pm \sqrt{M_1^2 + M_2^2 + 2M_1 M_2 \cos ka} \right)$$

(b) Discuss the form of the dispersion relation and the nature of the normal modes when $M_1 \gg M_2$. (i.e. calculate and draw the normal modes)

(c) Compare the dispersion relation with that of the monatomic linear chain when $M_1 \sim M_2$. (i.e. calculate and draw the normal modes)

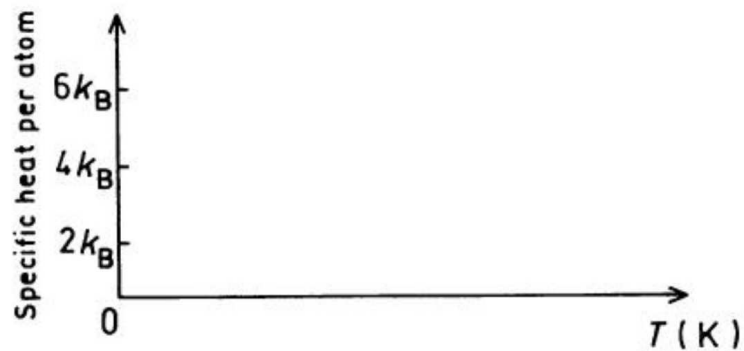
2. Specific heat (20)

Hljóðeiginleikar rafsvara yfirgnæfa varmahegðan og aðra eiginleika eins og ljósleiðni. Demantur er einnar atóma rafsvari úr kolefni sem hefur 10^{21} atoms/cm⁻³.

- Rissaðu, varmarýmd (á atóm) sem fall af hitastigi.
- Hvernig er T_{Debye} tengt Debye tíðninni ω_D ?
- Ef að hljóðhraðinn við lágar tíðnir er 5×10^5 m/s, hvað er þá góð nálgun fyrir ω_D ?

Acoustic properties of dielectric solids dominate their thermodynamic behavior and other properties such as photoconducting resistance. Diamond is a monoatomic dielectric solid of carbon having 10^{23} atoms/cm⁻³.

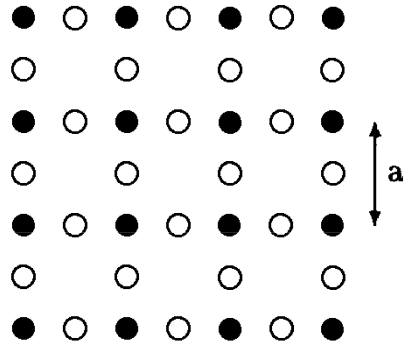
- Sketch, roughly, its specific heat (per atom) as a function of absolute temperature.
- How is T_{Debye} related to the Debye frequency ω_D ?
- If the acoustic velocity at low frequencies is 5×10^5 m/s, what is approximately the value of ω_D ?



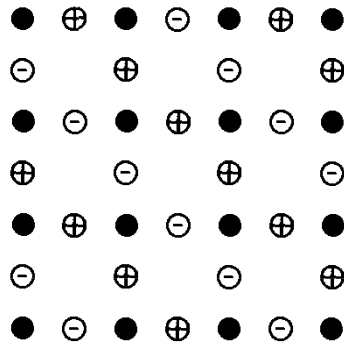
(Próf maí 2016)

3. Copper Oxide Layers (15)

The common building blocks for most high temperature (high T_c) superconductors are copper oxide layers, as shown below. Assume the distance between copper atoms (filled circles) is a . For simplicity let us also assume that in the third dimension these CuO_2 layers are simply stacked with spacing c , and there are no other atoms in the crystal. In first approximation the layers have a four-fold symmetry; the crystal is tetragonal.



(a) Sketch the Bravais lattice and indicate a possible set of primitive vectors for this crystal. What is the unit cell, and what is the basis ?



(b) In LaCuO_4 one discovers, at closer inspection, that the CuO_2 lattice is actually not flat, but that the oxygen atoms are moved a small amount out of the plane (“up” or “down”) in an alternating fashion (in the figure a $+$ means up and a $-$ means down). What is the primitive cell and lattice spacing for this crystal ? What is the reciprocal lattice ? Describe (qualitatively) what happens in the X-ray diffraction pattern as the distortion is decreased gradually to zero.

LaCuO_4 is an antiferromagnetic insulator. High temperature superconductivity was discovered in a closely related compound $\text{La}_{1-x}\text{Ba}_x\text{CuO}_4$. See J. G. Bednorz and K. A. Müller, *Z. Physik B* **64**, 189 (1986).