Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Vor 2016

# Eðlisfræði þéttefnis I

**Lokapróf**

**20. Maí 2016 kl. 09:00 - 12:00**

Leyfileg hjálpargögn eru skriffæri, vasareiknir og eintak af kennslubók(um) (ein eða fleiri):

- Harald Ibach and Hans Lüth, *Solid-State Physics: An Introduction to Principles of Materials Science*, 4th ed., Springer-Verlag, 2009
- Steven H. Simon, *The Oxford Solid State Basics*, Oxford University Press, 2013
- Charles Kittel, *Introduction to Solid State Physcis*, John Wiley & Sons
- Neil W. Ashcroft and N. David Mermin, *Solid State Physics*, Brooks Cole, 1976
- M. Ali Omar, Elementary Solid State Physics: Principles and Applications, Addison-Wesley

en engar glósur eða dæmi.

1. **X-ray diffraction** (15)

Þegar þú situr fyrir framan gamla litasjónvarpið með 25 kV mætti á myndlampanum þá eru miklar líkur á að þú verðir fyrir Röntgengeislun.

(a) Hvað er það sem veldur mestu flæði Röntgengeisla ?

(b) Fyrir samfelldu dreifinguna sem fram kemur, reiknaðu stystu bylgjulengd (hæsta orka) Röntgengeislanna.

(c) Fyrir salt (NaCl) kristall sem komið er fyrir framan við myndlampann, reiknaðu Bragg horn fyrstu gráðu speglunar við $\lambda = 0.5$ Å. ($\rho_{\mathrm{NaCl}} = 2.165$ g/cm$^3$ og $M = 58.45$ g/mol).

When sitting in front of a tube color TV with a 25 kV picture tube potential you have an excellent chance of being irradiated with X-rays.

(a) What process produces most of the X-ray flux ?

(b) For the resulting continous distribution, calculate the shortest wavelength (maximum energy) X-ray.

(c) For a rock salt (NaCl) crystal placed in front of the tube, calculate the Bragg angle for a first order reflection maximum at $\lambda = 0.5$ Å. ($\rho_{\mathrm{NaCl}} = 2.165$ g/cm$^3$ and $M = 58.45$ g/mol)

2. **Einnar atóma keðja – Monatomic chain** (18)

Gera skal ráð fyrir einnar atóma keðju þar sem bæði er víxlverkun milli næstu granna og þar næstu granna. Táknum gormstuðul milli næstu granna með $K_1$ og milli þarnæstu granna með $K_2$, massa atómsins með $M$, og grindarfastann með $a$.

(a) Rita skal hreyfijöfnur fyrir atómin og finna titringstíðni grindarinnar $\omega(k)$.

(b) Hver er hljóðhraðinn fyrir þessa keðju ?

Consider a monatomic chain which have both the nearest-neighbor and second nearest-neighbor interaction between atoms. Let us denote the nearest-neighbor spring constant by $K_1$, the second nearest-neighbor spring constant by $K_2$, the mass of the atoms by $M$, and the lattice constant by $a$.

(a) Write down the equation of motion for the atoms and solve for the lattice vibrational frequencies $\omega(k)$.

(b) What is the velocity of sound for this chain ?

3. **Bravais grind – Bravais lattice** (10)

Ef gefið er að grunnvigrar grindar séu $\mathbf{a}(a/2)(\mathbf{i}+\mathbf{j})$, $\mathbf{b}(a/2)(\mathbf{j}+\mathbf{k})$, og $\mathbf{c}(a/2)(\mathbf{k}+\mathbf{i})$, þar sem $\mathbf{i}$, $\mathbf{j}$ og $\mathbf{k}$ eru þessir venjulegu einingavigrar í Kartesíusarhnitum, hver er þá Bravais grindin ?

Given that the primitive basis vectors of a lattice are $\mathbf{a}(a/2)(\mathbf{i}+\mathbf{j})$, $\mathbf{b}(a/2)(\mathbf{j}+\mathbf{k})$, and $\mathbf{c}(a/2)(\mathbf{k}+\mathbf{i})$, where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the usual three unit vectors along cartesian coordinates, what is the Bravais lattice ?

4. **Fermi level adjustment in silicon** (10)

Kísilsýni við 300 K er íbætt með rafþega íbót af þéttleika $N_\mathrm{A} = 5 \times 10^{16}$ cm$^{-3}$. Ákvarða íbótarþéttleika rafgjafa íbótar sem bæta verður við þannig að kísillinn verði $n-$leiðandi og Fermi orkustigið sé 0.12 eV neðan við leiðniborðabrún.

A silicon sample at 300 K contains an acceptor impurity concentartion of $N_\mathrm{A} = 5 \times 10^{16}$ cm$^{-3}$. Determine the concentration of donor impurity atoms that must be added so that the silicon is $n-$type and the Fermi level is 0.12 eV below the conduction band edge. Virkur ástandsþéttleiki leiðniborða kísils er $N_\mathrm{C} = 2.86 \times 10^{19}$ cm$^{-3}$.

5. **Specific heat** (16)

Hljóðeiginleikar rafsvara yfirgnæfa varmahegðan og aðra eiginleika eins og ljósleiðni. Demantur er einnar atóma rafsvari úr kolefni sem hefur $10^{21}$ atoms/cm$^{-3}$.

(a) Rissaðu, varmarýmd (á atóm) sem fall af hitastigi.

(b) Hvernig er $T_{\text{Debye}}$ tengt Debye tíðninni $\omega_{\text{D}}$ ?

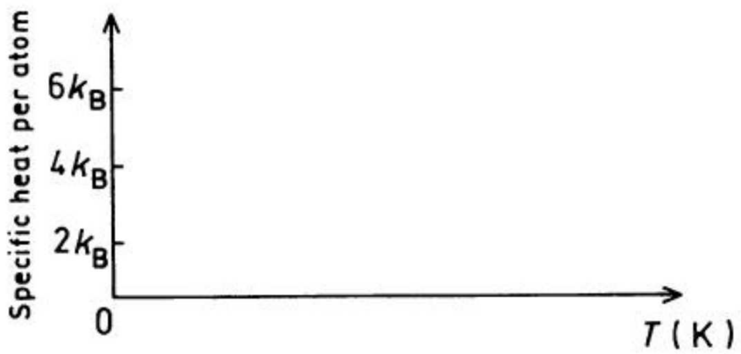
(c) Ef að hljóðhraðinn við lágar tíðnir er $5 \times 10^5$ cm/s, hvað er þá góð nálgun fyrir $\omega_{\text{D}}$ ?

Acoustic properties of dielectric solids dominate their thermodynamic behavior and other properties such as photoconducting resistance. Diamond is a monoatomic dielectric solid of carbon having $10^{21}$ atoms/cm$^{-3}$.

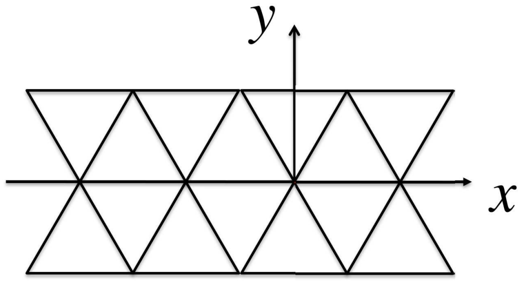
(a) Sketch, roughly, its specific heat (per atom) as a function of absolute temperature.

(b) How is $T_{\text{Debye}}$ related to the Debye frequency $\omega_{\text{D}}$ ?

(c) If the acoustic velocity at low frequencies is $5 \times 10^5$ cm/s, what is approximately the value of $\omega_{\text{D}}$ ?



6. **Two-dimensional triangular lattice – reciprocal lattice** (10)




(a) Merkið inn grunngrindareiningu í þessari tvívíðu þríhyrningsgrind. Finnið grunn vigranna.

(b) Finnið grunn vigra nykurgrindarinnar.

(a) Identify the primitive unit cell of a two-dimensional triangular lattice. Find the basis vectors.

(b) Construct the basis vectors of the reciprocal unit cell.

7. **Resistivity of metal** (5)

Rissaðu upp eðlisviðnám málms sem fall af hitastigi frá 0 K upp í 800 K.

Sketch the resistivity of metal as a function of temperature in the range 0 – 800 K.

8. **Intrinsic semiconductor** (16)

Gerum ráð fyrir eiginleiðandi hálfleiðara. Látum $\mathcal{E}$ vera orku rafeindar. Látum $g_{\mathrm{C}}(\mathcal{E})$ vera ástandsþéttleika í leiðniborðanum, og $g_{\mathrm{V}}(\mathcal{E})$ vera ástandsþéttleika í gildisborðanum. Gerum ráð fyrir að $\mathcal{E}_{\mathrm{C}} - \mathcal{E}_{\mathrm{F}} \gg k_{\mathrm{B}}T$, $\mathcal{E}_{\mathrm{F}} - \mathcal{E}_{\mathrm{V}} \gg k_{\mathrm{B}}T$, og

$$g_{\mathrm{C}}(\mathcal{E}) = C_1(\mathcal{E} - \mathcal{E}_{\mathrm{C}})^{1/2}$$

$$g_{\mathrm{V}}(\mathcal{E}) = C_2(\mathcal{E}_{\mathrm{V}} - \mathcal{E})^{1/2}$$

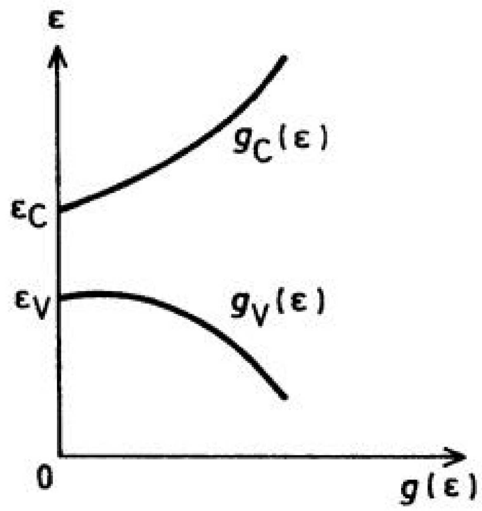
þar sem $\mathcal{E}_{\mathrm{C}}$ táknar orku við lágmark leiðniborða og $\mathcal{E}_{\mathrm{V}}$ orku við hámark gildisborða. Fermi orkan er $\mathcal{E}_{\mathrm{F}}$.

(a) Finna skal jöfnu fyrir $n$, rafeindaþéttleika í leiðniborðanum, sem fall af $k_{\mathrm{B}}$, $T$, $C_1$, $\mathcal{E}_{\mathrm{C}}$, $\mathcal{E}_{\mathrm{F}}$ og einingarlausu ákveðnu tegri.

(b) Finna skal jöfnu fyrir $p$, holuþéttleika í gildisborðanum, sem fall af $k_{\mathrm{B}}$, $T$, $C_2$, $\mathcal{E}_{\mathrm{V}}$, $\mathcal{E}_{\mathrm{F}}$ og einingarlausu ákveðnu tegri.

(c) Finna nákvæma jöfnu fyrir $\mathcal{E}_{\mathrm{F}}(T)$.

(d) Hver eða engin, af niðurstöðunum í (a), (b) eða (c) gildir ef hálfleiðarinn er íbættur með rafgjafa atómum ? Útskýrið.




Consider an intrinsic semiconductor. Let $\mathcal{E}$ be the energy of an electron. Let $g_{\mathrm{C}}(\mathcal{E})$ be the densty of states in the conduction band, and $g_{\mathrm{V}}(\mathcal{E})$ be the density of states in the valence band. Assume $\mathcal{E}_{\mathrm{C}} - \mathcal{E}_{\mathrm{F}} \gg k_{\mathrm{B}}T$, $\mathcal{E}_{\mathrm{F}} - \mathcal{E}_{\mathrm{V}} \gg k_{\mathrm{B}}T$, and

$$g_{\mathrm{C}}(\mathcal{E}) = C_1(\mathcal{E} - \mathcal{E}_{\mathrm{C}})^{1/2}$$

$$g_{\mathrm{V}}(\mathcal{E}) = C_2(\mathcal{E}_{\mathrm{V}} - \mathcal{E})^{1/2}$$

where $\mathcal{E}_\mathrm{C}$ represents the energy of the bottom of the conduction band and $\mathcal{E}_\mathrm{V}$ the top of the valence band. The Fermi energy is $\mathcal{E}_\mathrm{F}$.

(a) Find an expression for $n$, the number of electrons in the conduction band, in terms of $k_\mathrm{B}$, $T$, $C_1$, $\mathcal{E}_\mathrm{C}$, $\mathcal{E}_\mathrm{F}$ and a dimensionless definite integral.

(b) Find an expression for $p$, the number of holes in the valence band, in terms of $k_\mathrm{B}$, $T$, $C_2$, $\mathcal{E}_\mathrm{V}$, $\mathcal{E}_\mathrm{F}$ and a dimensionless definite integral.

(c) Find an explicit expression for $\mathcal{E}_\mathrm{F}(T)$.

(d) Which, if any, of the results of (a), (b) or (c) remain true if the material is doped with donor atoms ? Explain.

# 1 Fastar

$$q = 1.602 \times 10^{-19} \quad \text{C}$$

$$\hbar = 1.0546 \times 10^{-34} \quad \text{Js}$$

$$m_{\text{e}} = 9.1096 \times 10^{-31} \quad \text{Js}$$

$$N_{\text{Av}} = 6.022 \times 10^{23} \quad \text{sameindir/mól}$$

$$k_{\text{B}} = 1.38 \times 10^{-23} \text{J/K} = 8.62 \times 10^{-5} \quad \text{eV/K}$$

$$\epsilon_0 = 8.854 \times 10^{-14} \quad \text{F/cm}$$

$$\epsilon_{\text{ox}}/\epsilon_0 = 3.9$$

$$\epsilon_{\text{Si}}/\epsilon_0 = 11.9$$

$$\epsilon_{\text{Ge}}/\epsilon_0 = 16$$

$$\epsilon_{\text{GaAs}}/\epsilon_0 = 13.1$$

Fyrir kísil við stofuhita:

$$n_{\text{i}} = 9.65 \times 10^{9} \quad \text{cm}^{-3}$$

Fyrir GaAs við stofuhita:

$$n_{\text{i}} = 2.25 \times 10^{9} \quad \text{cm}^{-3}$$

# 2 Hálfleiðarar

$$E_{\text{H}} = -\frac{m_{\text{e}} q^4}{8\epsilon_0^2 h^2 n^2} = -\frac{13.6}{n^2}$$

$$E_{\text{g}} = 1.17 - \frac{(4.73 \times 10^{-4})T^2}{(T + 636)} \qquad \text{kísill}$$

$$E_{\text{g}} = 1.52 - \frac{(5.4 \times 10^{-4})T^2}{(T + 204)} \qquad \text{GaAs}$$

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_{\text{F}}}{kT}\right)}$$

$$n = \int_{\infty}^{E_{\text{c}}} f(E) N(E) dE$$

$$N(E) = 4\pi \left(\frac{2m^*}{h^2}\right)^{3/2} E^{1/2}$$

$$f(E) \approx \exp\left(-\frac{E - E_{\text{F}}}{kT}\right) \text{ ef } E - E_{\text{F}} > 3kT$$

$$f(E) \approx 1 - \exp\left(-\frac{E_{\text{F}} - E}{kT}\right) \text{ ef } E - E_{\text{F}} < 3kT$$

$$n \approx N_{\text{c}} \exp\left(-\frac{E_{\text{c}} - E_{\text{F}}}{kT}\right)$$

$$N_{\text{c}} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$p \approx N_{\text{v}} \exp\left(-\frac{E_{\text{F}} - E_{\text{v}}}{kT}\right)$$

$$N_{\text{v}} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$np = N_{\text{c}} N_{\text{v}} \exp\left(-\frac{E_{\text{g}}}{kT}\right) = n_{\text{i}}^2$$

$$n = n_{\text{i}} \exp\left(\frac{E_{\text{F}} - E_{\text{i}}}{kT}\right)$$

$$p = n_{\text{i}} \exp\left(\frac{E_{\text{i}} - E_{\text{F}}}{kT}\right)$$

$$E_{\text{c}} - E_{\text{F}} = kT \ln\left(\frac{N_{\text{c}}}{N_{\text{D}}}\right)$$

$$E_{\text{F}} - E_{\text{v}} = kT \ln\left(\frac{N_{\text{v}}}{N_{\text{A}}}\right)$$

$$np = n_{\mathrm{i}}^2$$

Við stofuhita fyrir kísil

$$N_{\mathrm{c}} = 2.8 \times 10^{19} \quad \mathrm{cm}^{-3}$$

$$N_{\mathrm{v}} = 1.04 \times 10^{19} \quad \mathrm{cm}^{-3}$$

Við stofuhita fyrir GaAs

$$N_{\mathrm{c}} = 4.7 \times 10^{17} \quad \mathrm{cm}^{-3}$$

$$N_{\mathrm{v}} = 7 \times 10^{18} \quad \mathrm{cm}^{-3}$$

n-leiðandi hálfleiðari

$$n_{\mathrm{n}} = \frac{1}{2}\left[N_{\mathrm{D}} - N_{\mathrm{A}} + \sqrt{(N_{\mathrm{D}} - N_{\mathrm{A}})^2 + 4n_{\mathrm{i}}^2}\right]$$

og

$$p_{\mathrm{n}} = \frac{n_{\mathrm{i}}^2}{n_{\mathrm{n}}}$$

p-leiðandi hálfleiðari

$$p_{\mathrm{p}} = \frac{1}{2}\left[N_{\mathrm{A}} - N_{\mathrm{D}} + \sqrt{(N_{\mathrm{A}} - N_{\mathrm{D}})^2 + 4n_{\mathrm{i}}^2}\right]$$

og

$$n_{\mathrm{p}} = \frac{n_{\mathrm{i}}^2}{p_{\mathrm{p}}}$$

$$N_{\mathrm{C}} = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$N_{\mathrm{V}} = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$J = \sigma\mathcal{E}$$

$$\sigma = \frac{nq^2\tau}{m_{\mathrm{n}}^*} \qquad [\Omega\mathrm{cm}]^{-1}$$

$$\sigma = qn\mu_{\mathrm{n}}$$

$$\mu_{\mathrm{n}} = \frac{q\tau}{m_{\mathrm{n}}^*}$$

$$J = q(n\mu_{\mathrm{n}} + p\mu_{\mathrm{p}})\mathcal{E} = \sigma\mathcal{E}$$

$$R = \frac{\rho L}{Wd} = \frac{L}{Wd}\frac{1}{\sigma}$$

# 3 Viðnám

$$R = \frac{\rho L}{A}$$

$$\sigma = \frac{1}{\rho} = (q\mu_{\mathrm{n}} n + q\mu_{\mathrm{p}} p)$$

$$R = \frac{1}{G} = \frac{L}{W}\frac{1}{g}$$