Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2017

Eðlisfræði þéttefnis I

Dæmablað 6

Skilafrestur 5. October 2017 kl. 15:00

1. **Einnar atóma keðja – Monatomic chain** (15)

   Gera skal ráð fyrir einnar atóma keðju þar sem bæði er víxlverkun milli næstu granna og þar næstu granna. Táknum gormstuðul milli næstu granna með K_1 og milli þarnæstu granna með K_2, massa atómsins með M, og grindarfastann með a.

   (a) Rita skal hreyfijöfnur fyrir atómin og finna titringstíðni grindarinnar $\omega(k)$.

   (b) Hver er hljóðhraðinn fyrir þessa keðju ?

   Consider a monatomic chain which have both the nearest-neighbor and second nearest-neighbor interaction between atoms. Let us denote the nearest-neighbor spring constant by K_1, the second nearest-neighbor spring constant by K_2, the mass of the atoms by M, and the lattice constant by a.

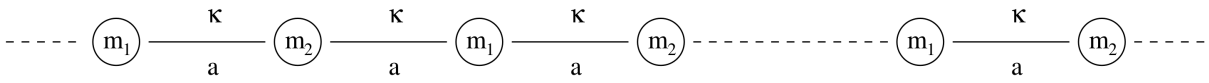
   (a) Write down the equation of motion for the atoms and solve for the lattice vibrational frequencies $\omega(k)$.

   (b) What is the velocity of sound for this chain ?

   (Próf maí 2016)

2. **Normal modes of a one dimensional diatomic crystal** (20)

Consider a straight chain of atoms with alternating mass m_1 and m_2 and interatomic distance a. Nearest neighbors interact through a spring of constant κ.



(a) Establish the dispersion relation for the normal modes of the chain.

(b) Discuss the cases $m_1 = m_2$ and $m_1 \gg m_2$ making use of reduced and extended zone representations.

3. **Dispersion relation** (15)

Consider 1D chain with identical masses M. Assume that there are nearest neighbor(nn) springs with spring constant K_1 and next nearest neighbor (nnn) springs with spring constant $K_2 < K_1$.

(a) Find the dispersion relation for this system.

(b) Calculate the speed of sound and compare to a system without nnn interactions.

(c) Now generalize to a new energy:

$$U^{\text{harmonic}} = \sum_n \sum_{m>0} K_m \left[u(na) - u([n+m]\,a)\right]^2$$

(i) Show that the dispersion relation is:

$$\omega = 2\sqrt{\sum_{m>0} K_m \frac{\sin^2(\frac{1}{2}mka)}{M}}$$

(ii) Show that the long wavelength limit of the dispersion relation for $K_m = K_1/m^p$ when $p = 3$ is

$$\omega \sim k\sqrt{|\ln k|}$$