Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2017

Eðlisfræði þéttefnis I

Dæmablað 8

Skilafrestur 19. Október 2017 kl. 15:00

1. **Líkan Einstein fyrir eðlisvarma – Einstein's model for specific heat** (15)

   Líkan Einstein fyrir þéttefni gefur jöfnu fyrir eðlisvarma

   $$C_{\mathrm{v}} = 3N_0 k \left(\frac{\theta_{\mathrm{E}}}{T}\right)^2 \frac{\exp(\theta_{\mathrm{E}}/T)}{(\exp(\theta_{\mathrm{E}}/T)-1)^2}$$

   þar sem $\theta_{\mathrm{E}} = hv_{\mathrm{E}}/k$. Stuðullinn θ_{E} er nefndur hið einkennandi hitastig. Sýna skal

   (a) að fyrir há hitastig fæst lögmál Dulong-Petit.

   (b) að fyrir mjög lág hitastig fæst ekki T^3 lögmálið.

   Einstein's model of solids gives the expression for the specific heat

   $$C_{\mathrm{v}} = 3N_0 k \left(\frac{\theta_{\mathrm{E}}}{T}\right)^2 \frac{\exp(\theta_{\mathrm{E}}/T)}{(\exp(\theta_{\mathrm{E}}/T)-1)^2}$$

   where $\theta_{\mathrm{E}} = hv_{\mathrm{E}}/k$. The factor θ_{E} is called the characteristic temperature. Show that

   (a) at high temperatures Dulong-Petit law is reproduced.

   (b) at very low temperatures the T^3 law is not given.

   (Próf desember 2016)

2. **Electron gas in two dimensions** (20)

We consider a two dimensional gas of electrons of mass m with N electrons confined to an area A so there is a density $n = N/A$ of electrons per unit area.

(a) Express the Fermi wave vector magnitude k_{F} and the Fermi energy $\mathcal{E}_{\mathrm{F}}$ in terms of n

(b) Express the density of levels $g(\mathcal{E})$

(c) Write the Sommerfeld expansion for n and conclude as to the relation between the chemical potential μ and the Fermi energy $\mathcal{E}_{\mathrm{F}}$

(d) Obtain a relation between μ and $\mathcal{E}_{\mathrm{F}}$ directly from the relation

$$n = \int_{-\infty}^{\infty} \mathrm{d}\mathcal{E} g(\mathcal{E}) f(\mathcal{E})$$

where $f(\mathcal{E})$ is the Fermi-Dirac occupation factor (Hint: proceed with the change of variable $x = \mathrm{e}^{-(\mathcal{E}-\mu)/k_{\mathrm{B}}T}$). Write your result to the limit $k_{\mathrm{B}}T \ll \mathcal{E}_{\mathrm{F}}$.

(e) Comment on the difference between your answers to question (c) and (d).

3. **Thermal motion and resistivity** (10)

Show that if the random velocity of the electrons were due to thermal motion of a classical electron gas, the electrical resistivity would increase with the temperature as $T^{3/2}$.

4. **Second Neighbor Diatomic Chain** (10)

Consider a diatomic chain. In addition to the spring constant κ between neighboring masses, suppose that there is also a next nearest-neighbor coupling with spring constant κ' connecting equivalent masses in adjacent unit cells. Determine the dispersion relation for this system. What happens if $\kappa \gg \kappa'$?

5. **Fermi velocity** (10)

Estimate the ratio of the drift velocity to the Fermi velocity for a 2 mm^2 Cu wire carrying a 20 A current.