

# Eðlisfræði þéttfnis I

## Dæmablað 9

Skilafrestur 26. Október 2017 kl. 15:00

### 1. Anomalous density of states (15)

A certain two-dimensional simple-square lattice of lattice constant  $a$  has the dispersion relation  $(\omega_L(q))^2 = c_L^2 q^2$ , for longitudinal vibrations and  $(\omega_T(q))^2 = c_T^2 q^2$  for transverse vibrations for  $qa \ll 1$ . The density of mode frequencies  $D(\omega)$  determines the temperature dependence of the specific heat.

(a) What is  $D(\omega)$  for an  $N$ -atom crystal at frequencies  $\omega$  described by the dispersion relation above? That is, what is the number of modes between  $\omega$  and  $\omega + d\omega$ ? Suppose that for some wave-vector  $q_0$ , the  $\omega_L$  has an absolute maximum. Near this maximum  $\omega_L(q) = \omega_0 - A(q - q_0)^2$ . You can assume  $\omega_T(q)$  always lies well below  $\omega_0$ , so that these modes don't contribute to (b).

(b) Find the form of  $D(\omega)$  when  $\omega \leq \omega_0$ , and when  $\omega \geq \omega_0$ .

### 2. van Hove singularities (15)

(a) In a linear harmonic chain with only nearest-neighbor interactions, the normal-mode dispersion relation has the form  $\omega(q) = \omega_0 |\sin(qa/2)|$ , where the constant  $\omega_0$  is the maximum frequency (assumed when  $q$  is on the zone boundary). Show that the density of normal modes in this case is given by

$$D(\omega) = \frac{2}{\pi a \sqrt{\omega_0^2 - \omega^2}}.$$

The singularity at  $\omega_0 - \omega$  is called a van Hove singularity.

(b) In three dimensions the van Hove singularities are infinities not in the normal mode density itself, but in its derivative. Show that the normal modes in the neighborhood of a maximum of  $\omega(q)$ , for example, lead to a term in the normal mode density that varies as  $(\omega_0 - \omega)^{1/2}$ .

### 3. Sound waves (10)

Would you expect to find sound waves in small molecules? If not, how do you explain the propagation of sound waves in gaseous substances?

### 4. Fermi Surface Shapes (10)

Consider a divalent atom, such as Ca or Sr, that forms an fcc lattice (with a single atom basis). In the absence of a periodic potential, would the Fermi surface touch the Brillouin zone boundary? What fraction of the states in the first Brillouin zone remain empty?

### 5. Neutron Scattering (10)

The conventional unit cell dimension for a particular bcc solid is 0.24 nm. Two orders of diffraction are observed. What is the minimum energy of the neutrons? At what temperature would such neutrons be dominant if the distribution is Maxwell-Boltzmann.