Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2018

Eðlisfræði þéttefnis I

Dæmablað 10

Skilafrestur 11. Nóvember 2018 kl. 15:00

1. **Conduction mechanism of a solid** (15)

Myndin hér að neðan sýnir eðlisviðnám þéttefnis sem fall af hitastigi ($ErRhB_4$ óhreint, en ekki viljandi skemmt).

(a) Útfrá grafinu segið hvort þéttefnið er málmur eða einangrari.

(b) Lýsið megin ferlunum sem stýra leininni á eftirfarandi hitastigsbilum:

(1) T mjög nærri 0 K

(2) T nærri 25 K

(3) T nærri 300 K

(c) Áætlið meðal frjálsa vegalengd og meðal frjálsan tíma við $T = 0$ K og $T = 300$ K. Er þetta góður málmur ?

Nota má eftrifarandi gildi: $n = 10^{23}$ cm^{-3} og $v_{\mathrm{F}} = 10^8$ cm/s.

The figure below shows a rough plot of the electrical resistance of a solid ($ErRhB_4$ impure, but not purposely damaged).

(a) From the graph, is this material a metal or an insulator ?

(b) Describe the principal physical processes that account for the following three temperature regions:

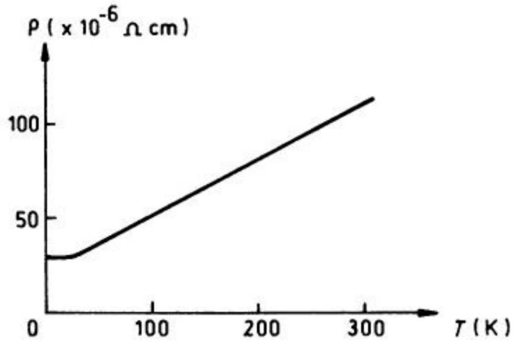
(1) T very near 0 K

(2) T near 25 K

(3) T near 300 K

(c) Estimate the mean free path and mean free time at $T = 0$ K and $T = 300$ K. Is the material a good metal ?

Useful parameters: $n = 10^{23}$ cm^{-3} og $v_{\mathrm{F}} = 10^8$ cm/s.



(Próf desember 2015)

2. **Conductivity of a metal** (10)

Notið jöfnuna

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = -e\mathbf{E}$$

fyrir skriðþunga rafeinda til að finna AC leiðni málms. Svar þitt skal vera á forminu $\sigma(\omega, n, e, \tau)$.

Use the equation

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = -e\mathbf{E}$$

for the electron momentum to find the AC conductivity of a metal. Your answer should be of the form $\sigma(\omega, n, e, \tau)$.

(Próf desember 2015)

3. **Electron gas in two dimensions** (20)

For a free and independent electron gas in two dimensions

(a) What is the relation between n and k_{F} in two dimensions ?

(b) Prove that in two dimensions the free electron density of levels $D(E)$ is a constant independent of E for $E > 0$, and 0 for $E < 0$. What is the constant ?

(c) Show that because $D(E)$ constant, every term in the Sommerfeld expansion for n vanishes except the $T = 0$ term. Deduce that $\mu = E_{\mathrm{F}}$ at any temperature.

(d) Deduce from

$$n = \int_{-\infty}^{\infty} dE D(E) f(E)$$

that when $D(E)$ is as in (c), then

$$\mu + k_{\mathrm{B}} T \ln\left(1 + \exp(-\mu/k_{\mathrm{B}} T)\right) = E_{\mathrm{F}}$$

(e) Estimate from the above equation the amount by which μ differs from E_{F}. Comment on the numerical significance of this "failure" of Sommerfeld expansion, and on the mathematical reason for the "failure".

4. **The Kronig-Penney model** (20)

Consider an electron in 1D in the presence of the periodic potential (Kronig-Penney model)

$$U(x) = \sum_{m=-\infty}^{\infty} U_0 \Theta(x - ma)\Theta(ma + b - x)$$

(a) Restrict your attention to a single unit cell, and write down the boundary conditions for the wave function as required by Bloch's theorem.

(b) Solve the Schrödinger equation by constructing ψ(x) from plane waves and imposing suitable boundary conditions at $x = 0, b, a$. The results is a relation between the Bloch index k and the energy.

(c) Take the limit $b \longrightarrow 0$, $U_0 \longrightarrow \infty$ with $U_0 b \longrightarrow W_0 \frac{\hbar^2 a^{-2}}{m}$. Show that the condition for the Bloch index simplifies to

$$\cos(ka) = \frac{W_0}{qa}\sin(qa) + \cos(qa)$$

where q is related to the eigenenergy $\mathcal{E}$ via $q = (2m\mathcal{E}/\hbar^2)^{1/2}$.

(d) Produce plots of the lowest two energy bands $\mathcal{E}_{nk} (n = 0, 1)$ in the limit of part (c) with $a = 1$, $m = 1$, $\hbar = 1$, and $W_0 = 0.5$.

4. **Periodic potential** (10)

Gerum ráð fyrir einvíðu rafeindakerfi sem hlítir veiku lotubundnu mætti

$$U(x) = U_0 \left[\cos^4\left(\frac{\pi x}{a}\right) - \frac{3}{8}\right]$$

Ákvraðið margfeldni punktana við $k = \pm\pi/a$ og $k = \pm 2\pi/a$. Finnið og teiknið tvístrunarsambandið í fyrsta Brillouin svæðinu. Teiknið orkuna í einingunni $\hbar^2\pi^2/2ma^2$, bylgjuvigur í einingunni $1/a$, og gerið ráð fyrir að $U_0 = 0.1$ í þessum einingum.

Consider a one dimensional electron system subject to a weak periodic potential

$$U(x) = U_0 \left[\cos^4\left(\frac{\pi x}{a}\right) - \frac{3}{8}\right]$$

Determine the degeneracy points at $k = \pm\pi/a$ and $k = \pm 2\pi/a$. Find and plot the dispersions of energy bands in the first Brillouin zone. Plot the energies in units of $\hbar^2\pi^2/2ma^2$, the wave numbers in units of $1/a$, and assume that $U_0 = 0.1$ in these units.

(Próf desember 2016)

5. **Fermi level adjustment in silicon** (10)

Kísilsýni við 300 K er íbætt með rafþega íbót af þéttleika $N_\mathrm{A} = 5 \times 10^{16}$ cm^{-3}. Ákvarða íbótarþéttleika rafgjafa íbótar sem bæta verður við þannig að kísillinn verði $n-$leiðandi og Fermi orkustigið sé 0.12 eV neðan við leiðniborðabrún. Virkur ástandsþéttleiki leiðniborða kísils er $N_\mathrm{C} = 2.86 \times 10^{19}$ cm^{-3}.

A silicon sample at 300 K contains an acceptor impurity concentartion of $N_\mathrm{A} = 5 \times 10^{16}$ cm^{-3}. Determine the concentration of donor impurity atoms that must be added so that the silicon is $n-$type and the Fermi level is 0.12 eV below the conduction band edge.

(Próf maí 2016)