Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2018

Eðlisfræði þéttefnis I

Dæmablað 7

Skilafrestur 16. October 2018 kl. 15:00

1. **Dispersion relation** (15)

   Consider a linear chain in which alternate ions have mass M_1 and M_2, and only nearest neighbors interact.

   (a) Show that the dispersion relation for normal modes is

   $$\omega^2 = \frac{K}{M_1 M_2}\left(M_1 + M_2 \pm \sqrt{M_1^2 + M_2^2 + 2M_1 M_2 \cos ka}\right)$$

   (b) Discuss the form of the dispersion relation and the nature of the normal modes when $M_1 \gg M_2$. (i.e. calculate and draw the normal modes)

   (c) Compare the dispersion relation with that of the monatomic linear chain when $M_1 \sim M_2$. (i.e. calculate and draw the normal modes)

2. **Specific heat** (20)

   Hljóðeiginleikar rafsvara yfirgnæfa varmahegðan og aðra eiginleika eins og ljósleiðni. Demantur er einnar atóma rafsvari úr kolefni sem hefur 10^{21} atoms/cm^{-3}.

   (a) Rissaðu, varmarýmd (á atóm) sem fall af hitastigi.

   (b) Hvernig er T_{Debye} tengt Debye tíðninni ω_{D} ?

   (c) Ef að hljóðhraðinn við lágar tíðnir er 5×10^5 m/s, hvað er þá góð nálgun fyrir ω_{D} ?

Acoustic properties of dielectric solids dominate their thermodynamic behavior and other properties such as photoconducting resistance. Diamond is a monoatomic dielectric solid of carbon having 10^{23} atoms/cm^{-3}.

(a) Sketch, roughly, its specific heat (per atom) as a function of absolute temperature.

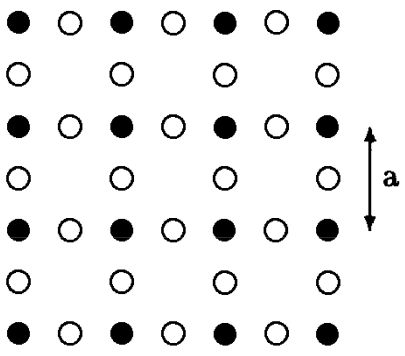
(b) How is T_{Debye} related to the Debye frequency ω_{D} ?

(c) If the acoustic velocity at low frequencies is 5×10^5 m/s, what is approximately the value of ω_{D} ?

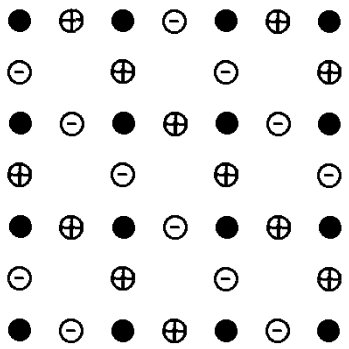
(Próf maí 2016)

3. **Copper Oxide Layers** (15)

The common building blocks for most high temperature (high T_{c}) superconductors are copper oxide layers, as shown below. Assume the distance between copper atoms (filled circles) is a. For simplicity let us also assume that in the third dimension these CuO_2 layers are simply stacked with spacing c, and there are no other atoms in the crystal. In first approximation the layers have a four-fold symmetry; the crystal is tetragonal.



(a) Sketch the Bravais lattice and indicate a possible set of primitive vectors for this crystal. What is the unit cell, and what is the basis ?



(b) In $LaCuO_4$ one discovers, at closer inspection, that the CuO_2 lattice is actually not flat, but that the oxygen atoms are moved a small amount out of the plane ("up" or "down") in an alternating fashion (in the figure a + meand up and a - means down). What is the primitive cell and lattice spacing for this crystal ? What is the reciprocal lattice ? Describe (qualitatively) what happens in the X-ray diffraction pattern as the distortion is decreased gradually to zero.

$LaCuO_4$ is an antiferromagnetic insulator. High temperature superconductivity was discovered in a closely related compound $La_{1-x}Ba_xCuO_4$. See J. G. Bednorz and K. A. Müller, *Z. Physik B* **64**, 189 (1986).

4. **Varmarýmd $d-$víðs einangrara – Specific heat of a $d-$dimensional insulator** (15)

Gerum ráð fyrir $d-$víðum kristalli með tvístrunarsamband gefið sem $\omega = Ak^\lambda$ þar sem A og λ eru fastar. Setjum N sem fjölda grindarpunkta í sýninu.

(a) Reiknið hneppishraðann sem fall af k.

(b) Ef Debye hitastigið Θ_D er í réttu hlutfalli við N^α finnið þá α sem fall af λ og d.

(c) Ef ástandsþéttleiki hljóðeinda $D(\omega)$ er í réttu hlutfalli við ω^β þá skal finna β sem fall af λ og d.

(d) Ef að varmarýmdin C við lág hitastig er í réttu hlutfalli við T^δ finnið δ sem fall af λ og d. Ræðið niðurstöðurnar fyrir línulegt tvístrunarsamband með $d = 1$, 2, and 3.

Consider a $d-$dimensional crystal with the dispersion relation given as $\omega = Ak^{\lambda}$ where A and λ are constants. Let N be the number of lattice points in the sample.

(a) Calculate group velocity in terms of k.

(b) If the Debye temperature Θ_{D} is proportional to N^{α}. Calculate α in terms of λ and d.

(c) If the phonon density of states $D(\omega)$ is proportional to ω^{β} calculate β in terms of λ and d.

(d) If the heat capacity C at low temperatures is proportional to T^{δ}. Calculate δ in terms of λ and d. Discuss your results for the particular case of linear dispersion relation, with $d = 1$, 2, and 3.

(Próf Desember 2017)