1. Vetnisafhleðsla

Margfeldni $2^2P_{1/2}$ og $2^2P_{3/2}$ eru 2 og 4, en orkumunur milli þeirra og $1^2S_{1/2}$ er u.þ.b. sá sami.

Þess vegna er styrkhlutfall lína

$$\frac{2^2P_{1/2} \to 1^2S_{1/2}}{2^2P_{3/2} \to 1^2S_{1/2}} = \frac{1}{2}$$

2. Electron in a Thomson atom

Consider an electron oscillating along a diameter. When at a distance r from the center of the atom, the force acting on the electron is

$$F = \frac{1}{4\pi\varepsilon_0} \frac{4\pi}{3} r^3 \rho \frac{e}{r^2}$$

where

$$\rho = \frac{e}{\frac{4\pi R^3}{3}} > 0$$

since the net charge on atom-electron is $+e$. Therefore,

$$F = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{R^3} r$$

This force is attractive, i.e

directed toward the center of the atom. Hence, since

$$F = ma$$

we have

$$-\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2}r = m\ddot{r}$$

and

$$\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mR^3}$$

If the electron revolves in a circular orbit of radius R then $F = ma$

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{R^2} = m\frac{v^2}{R} = m\frac{(R\omega)^2}{R} = mR\omega^2$$

which gives

$$\omega^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{mR^3}$$

The two frequencies are seen to be equal. The equality applies also to oscillations of amplitude less than R and circular orbits of radius less than R, since the charge exterior to the amplitude or radius exerts zero force on the electron for spherically asymmetric charge distributions.

3. speed of a proton in a hydrogen atom

The periods of revolution of electron and proton are equal

$$\frac{2\pi r_e}{V_e} = \frac{2\pi r_p}{V_p}$$

we find

$$V_p = \frac{r_p}{r_e} V_e$$

The motion is about the center of mass of the electron-proton system, so that

$$m_p r_p = m_e r_e \Rightarrow \frac{r_p}{r_e} = \frac{m_e}{m_p}$$

and therefore

$$V_p = \frac{m_e}{m_p} V_e = \frac{m_e}{m_p} \frac{c}{137}$$

and

$$V_p = \frac{1}{1836} \frac{3\times10^8}{137} = 1.2\times10^3\ m/s$$

$\boxed{}$

4. Isotope shift

0.18 nm

$\boxed{}$

3.

(a) Tíðni fyrstu línu

$$\nu_1 = \frac{c}{\lambda_1} = cR_H\left(\frac{1}{m^2} - \frac{1}{(m+1)^2}\right)$$

Tíðni við útmörk raðar

$$\nu_\infty = \frac{c}{\lambda_\infty} = cR_H\left(\frac{1}{m^2} - 0\right)$$

Þar með er

$$\Delta\nu = \nu_\infty - \nu_1 = \frac{cR_H}{(m+1)^2}$$

(b)

$$\frac{\Delta\nu_{Ly}}{\Delta\nu_{Pf}} = \frac{cR_H/(1+1)^2}{cR_H/(5+1)^2} = 9$$

$\blacksquare$