1. Lyman alpha

Use

$$\frac{1}{\lambda} = Z^2 R \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

where R is the Rydberg constant 109678 cm^{-1}. For hydrogen

$$\frac{1}{\lambda} = R \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = 82258.5 \ cm^{-}$$

which corresponds to

$$\lambda = 121.6 \ nm$$

For helium

$$\frac{1}{\lambda} = 4R \left(\frac{1}{1^2} - \frac{1}{2^2} \right) = 329034 \ cm^{-1}$$

and

$$\lambda = 30.39 \ nm$$

☒

2. Radíi með 90% rafeindalíkur

Leysum fyrir R

$$\int_0^R |\psi_{1s}(r)|^2 4\pi r^2 \, dr = 0.9$$

eða

$$\int_R^\infty |\psi_{1s}(r)|^2 4\pi r^2 = 0.1$$

sem er auðveldara.

Þar með er

$$4\int_R^\infty r^2 e^{-2r} dr = e^{2R}(2R^2 + 2R + 1) = 0.1$$

Þar sem við notum

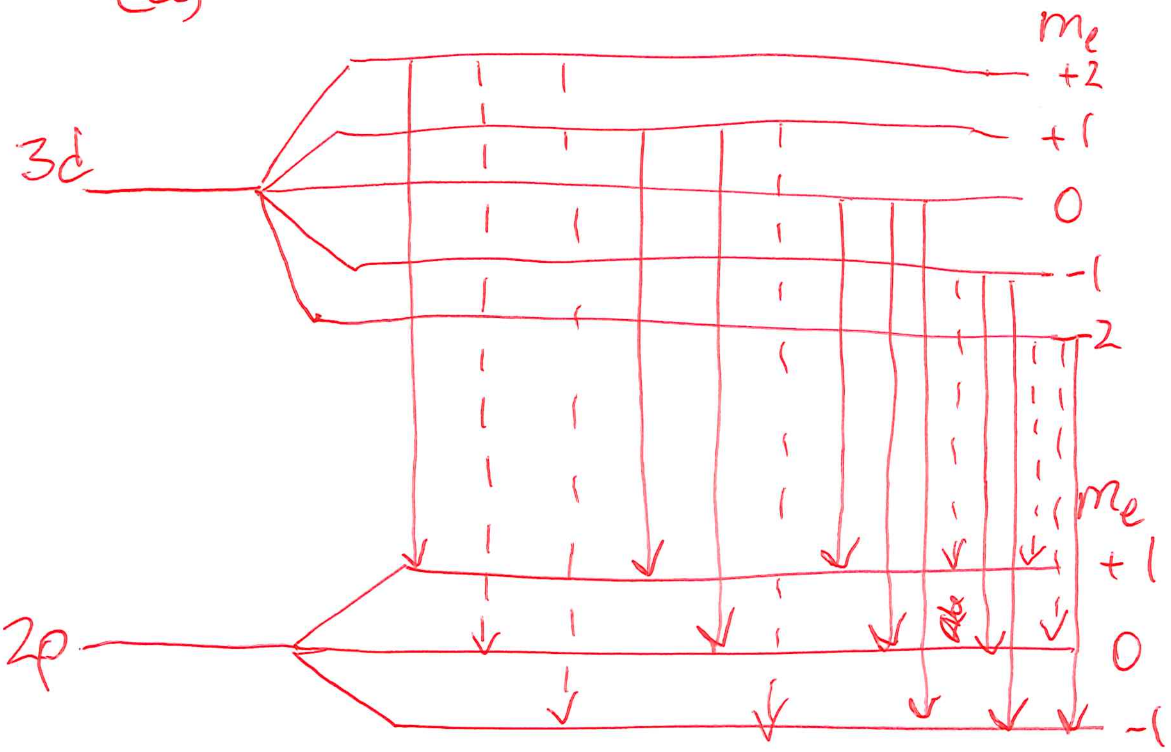
$$\int x^2 e^{ax} dx = e^{ax}\left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)$$

Leysum tölulega

$$R = 2.6612\, a_0 = 1.41\, \mathring{A}$$

4. Zeeman effect

(a)



(b) Transitions shown with dashed lines violate the $\Delta m_\ell = \pm 1$ selection rule

(c) The energy of the initial state is

$$E_i = E_{3d} + m_{\ell i} \Delta E$$

and the energy of the final state is

$$E_f = E_{2p} + m_{\ell f} \Delta E$$

(where ΔE is the splitting between ajacent states). The transition energies can be found from the energy difference

$$E_{3d} - E_{2p},\ E_{3d} - E_{2p} + \Delta E,$$

$$E_{3d} - E_{2p} + \Delta E$$

2 Ionized helium

(a) From $n=8$ level, downward transitions are possible to any level of smaller n. The transitions with the longest wavelengths are those with the smallest energy differences

$8 \to 7$

$$\Delta E = E_8 - E_7 = (-13.6\,eV)\,Z^2\left(\frac{1}{8^2} - \frac{1}{7^2}\right) = 0.260\,eV$$

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\ nm}{0.260\ eV} = 4.77\ \mu m$$

$8 \to 6$

$$\Delta E = E_8 - E_7 = 0.661\,eV$$

$$\lambda = 1.88\ \mu m$$

$8 \to 5$

$$\Delta E = E_8 - E_5 = 1.33\,eV$$

$$\lambda = 0.935\ \mu m$$

(b) The transition with the shortest wavelength is the one with the largest energy difference

$8 \to 1$

$$\Delta E = E_8 - E_1 = (-13.6\,eV) Z^2 \left(\frac{1}{8^2} - \frac{1}{1^2} \right) = 53{,}6\,eV$$

$$\lambda = \frac{1240\,nm}{53.6\,eV} = 23{,}2\,nm$$

(c) From the $n=8$ level, the atom can absorb a photon and the electron will jump to a state of larger n. The longest absorption wavelengths correspond to the smallest energy differences:

$8 \to 9$

$$\Delta E = E_9 - E_8 = (-13.6\,eV)\,Z^2\left(\frac{1}{9^2} - \frac{1}{8^2}\right) = 0{,}178\,eV$$

$$\lambda = 6.95\,\mu m$$

$8 \to 10$

$$\Delta E = E_{10} - E_8 = 0{,}306\,eV$$

$$\lambda = 4{,}05\,\mu m$$

$8 \to 11$

$$\Delta E = E_{11} - E_8 = 0.400\,eV$$

$$\lambda = 3{,}10\,\mu m$$

(d) The shortest absorption wavelength corresponds to the largest energy difference

$8 \to \infty$

$$\Delta E = E_\infty - E_8 = (-13.6\,eV)\,Z^2\left(0 - \frac{1}{8^2}\right) = 0.850\,eV$$

$$\lambda = 1.46\ \mu m$$

6. Helín atóm

Stigin eru

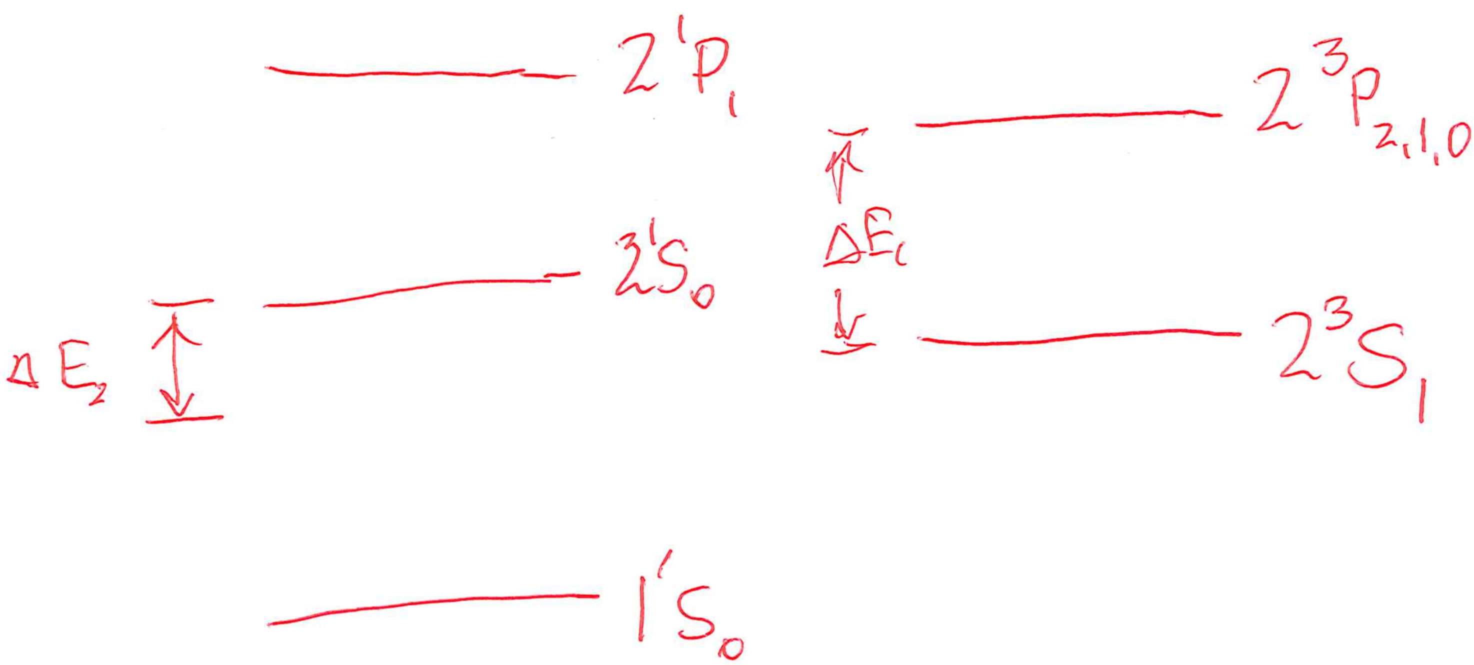
1^1S_0 eða $1s^2$

2^1S_0 eða $1s2s$

2^1P_1 eða $1s2p$

2^3S_1 eða $1s2s$

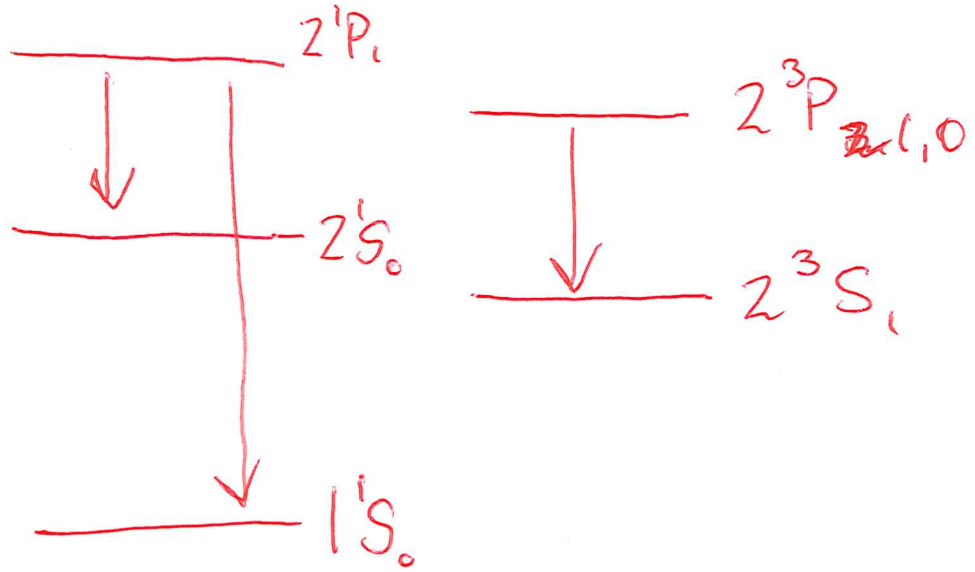
$2^3P_{2,1,0}$ eða $1s2p$



(b) ΔE_1 er orkumunur stiga með sama S. 3p stigin tilheyra 1s2p, sem hefur eina rafeind í 1s braut og hina í 2p braut. Sú síðari hefur hærri orku vegna skermunar kjarnhleðslunnar sem er meiri fyrir p rafeindina.

(c) ΔE_2 er orkumunur milli stiga með sama L í sömu rafeindaskipan en ólíkt S. Hún á rætur sínar að rekja til Coulomb víxverkunar.

(d) Leyfðar færslur



5. Rutherford dreifing

(a) diffurþversnið Rutherford er

$$\frac{d\sigma}{d\Omega} = \left(\frac{zZe^2}{2mv^2}\right)^2 \left(\sin\frac{\theta}{2}\right)^{-4}$$

Fyrir róteindir er

$$\beta = \frac{v}{c} = \frac{pc}{\sqrt{m^2c^4+p^2c^2}} = \frac{200}{\sqrt{938^2+200^2}} = 0{,}2085$$

einnig

$$\frac{e^2}{mv^2} = r_0\left(\frac{m_e}{m}\right)\left(\frac{v}{c}\right)^{-2}$$

þar sem

$$r_0 = \frac{e^2}{m_e c^2} = 2.82\times10^{-13}\ \text{cm}$$

er sígildur radíi rafeinda

þess vegna við $\theta = 30^\circ$

$$\frac{d\sigma}{d\Omega} = \left(\frac{13}{2}\right)^2 r_0^2 \left(\frac{m_e}{m}\right)^2 \left(\frac{v}{c}\right)^{-4} \left(\sin\frac{\theta}{2}\right)^{-4}$$

$$= 5.27\times10^{-28}\times(\sin 15)^{-4}$$

$$= 1.18\times10^{-25}\ \mathrm{cm}^2/\mathrm{sr}$$

(b) Rúmhornið fyrir 1cm

$$d\Omega = \frac{\pi(0.01)^2}{2^2} = 7.85\times10^{-5}\ \mathrm{sr}$$

Fjöldi rófeinda sem er dreift á tímaeiningu

$$\delta n = n\left(\frac{\rho t}{27}\right)A_v\left(\frac{d\sigma}{d\Omega}\right)\delta\Omega$$

$$= 10^{12}\left(\frac{2.7\times0.01}{27}\right)\times6.02\times10^{23}\times1.18\times10^{-25}$$
$$\times7.85\times10^{-5} = 5.58\times10^{3}\ \mathrm{s}^{-1}$$

(c) Heildar dreifiþversnið Rutherford

$$\sigma_I = \int \frac{d\sigma}{d\Omega} d\Omega = 2\pi \int_{5^\circ}^{180^\circ} \left(\frac{Ze^2}{2mv^2}\right) \frac{\sin\theta}{\sin^4\frac{\theta}{2}} d\theta$$

$$= 8\pi \left(\frac{Ze^2}{2mv^2}\right)^2 \int_{5^\circ}^{180^\circ} \left(\sin\frac{\theta}{2}\right)^{-3} d\sin\frac{\theta}{2}$$

$$= 4\pi \left(\frac{Ze^2}{2mv^2}\right)^2 \left[\frac{1}{(\sin 2{,}5)^2} - 1\right]$$

$$= 3.47 \times 10^{-24}\ \text{cm}^2$$

(d) Fjöldi róteinda $> 5^\circ$

$$\delta n = n\left(\frac{gt}{27}\right) A_v \sigma_I = 2.09 \times 10^9\ s^{-1}$$

$\blacksquare$