1. Stern-Gerlach magnet

The deflecting force is

$$F = \mu_z \frac{dB_z}{dz},$$

where

$$\mu_z = g_s \mu_B m_s,$$

since $l=0$. If D is the deflection and F is constant

$$D = \frac{1}{2} a t^2 = \frac{1}{2} \frac{F}{m_e} \left(\frac{L}{v}\right)^2$$

where, L is the magnet length and v the speed of atoms. Therefore,

$$D = \frac{1}{2} \frac{(g_s \mu_B m_s)}{m_e} \frac{dB_z}{dz} \left(\frac{L}{v}\right)^2$$

so that

$$\frac{dB_z}{dz} = \frac{2m_e D v^2}{L^2 g_s \mu_B m_s}$$

For atoms emitted from the oven

$$\frac{1}{2} m_e v^2 = 2 k_B T$$

with $T = 1233\,K$.

Hence,

$$\frac{dB_z}{dz} = \frac{8 k_B T D}{L^2 g_s \mu_B m_s}$$

$$= \frac{8(1.38\times10^{-23})(1233)(0.0005)}{(0.5)^2\, 2\times 9.27\times10^{-24} \times \frac{1}{2}}$$

$$= 29\ T/m$$

2. Hydrogen atom in magnetic field

(a) The orbit and spin energies are $(g_\ell \mu_B m_\ell)B$ and $(g_s \mu_B m_s)B$

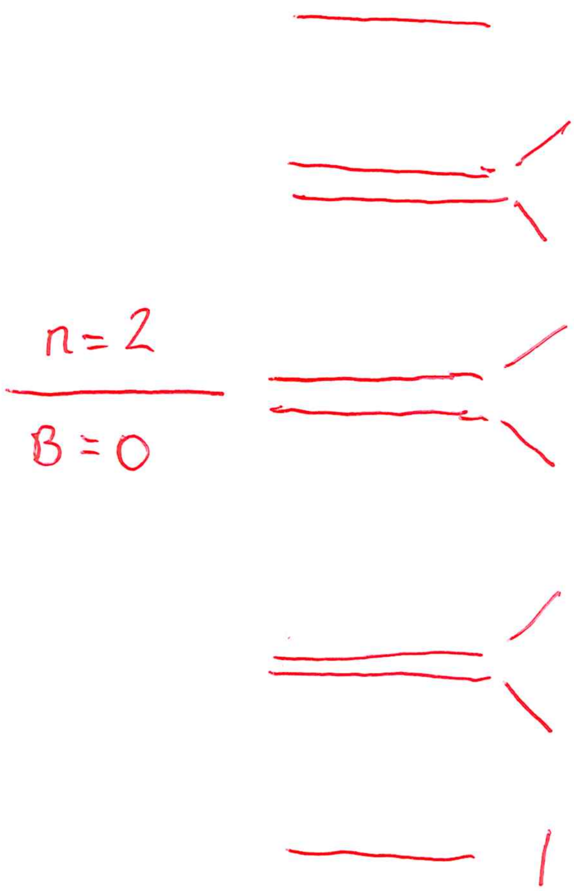
Hence, with respect to the energy for $B=0$,

$$\Delta E = g_\ell \mu_B m_\ell B + g_s \mu_B m_s B$$

$$= (g_\ell m_\ell + g_s m_s)\mu_B B = (m_\ell + 2m_s)\mu_B B$$

(b) For $n=2$, $\ell = 0, 1$

ℓ	m_ℓ	m_s	ΔE (units of $\mu_B B$)
0	0	$\frac{1}{2}$	+1
		$-\frac{1}{2}$	-1
	-1	$\frac{1}{2}$	0
		$-\frac{1}{2}$	-2
1	0	$\frac{1}{2}$	+1
		$-\frac{1}{2}$	-1
	+1	$\frac{1}{2}$	+2
		$-\frac{1}{2}$	0



l	m_l	m_s	E
1	1	$1/2$	+2
0	0	$1/2$	+1 ⊛
1	0	$1/2$	
1	−1	$1/2$	0 ⊛
1	1	$-1/2$	
0	0	$-1/2$	−1 ⊛
1	0	$-1/2$	
1	−1	$-1/2$	−2

⊛ two-fold degeneracy

c) The maximum seperation is

$$\Delta E_{max} = 4\mu_B B = 10.2 \text{ eV}$$

or

$$4\times(9.27\times10^{-24})B = (10.2)(1.6\times10^{-19})$$

or

$$B = 4.4\times10^{4}\ \text{T}$$

(1.) The ground state is spatially symmetric, so it goes with the symmetric (triplet) spin configuration. Thus the ground state is orthohelium and it is triply degenerate.

The excited states come in ortho and para (singlet) form; since the former go with the symmetric spiral wave function, the orthohelium states are higher in energy than the corresponding (nondegenerate) para states.

(b) The ground state and all excited states come ~~para~~ in both ortho and para form.

All are quadruply degenerate (or at any rate we have no way a priori of knowing whether ortho or para are higher in energy, since we don't know which goes with the symmetric spatial configuration.

3.

(a) Largest

$$j = 4 + 1/2 = 9/2$$

Largest

$$m_j = j = 9/2$$

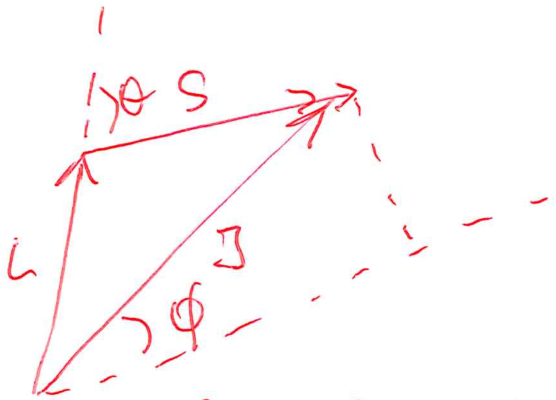
The magnitudes of the vectors are

$$J = \sqrt{j(j+1)}\,\hbar = \sqrt{99}\,\hbar/2$$

$$L = \sqrt{l(l+1)}\,\hbar = \sqrt{20}\,\hbar$$

$$S = \sqrt{s(s+1)}\,\hbar = \sqrt{3}\,\hbar/2$$

(b)



Apply the law of cosines to L, S, J triangle

$$J^2 = L^2 + S^2 - 2LS\cos(180 - \theta)$$

or

$$\frac{99}{4} = 20 + \frac{3}{4} + 2\sqrt{20 \times \frac{3}{4}}\cos\theta$$

so

$$\theta = \cos^{-1}\left(\frac{2}{\sqrt{15}}\right) = 58.91^\circ$$

Since $\underline{\mu}_l$ is antiparallel to $\underline{L}$ and $\underline{\mu}_s$ is antiparallel to $\underline{S}$, the angle between $\underline{\mu}_l$ and $\underline{\mu}_s$ is 58.91°

(c)

$$\cos\phi = \frac{J_z}{J} = \frac{9}{\sqrt{99}}$$

or

$$\phi = 25.24^\circ$$