1 Hydrogen in radio astronomy

(a) The energy levels of hydrogen in eV are

$$E_n = -\frac{13.6}{n^2}$$

for transitions between excited states $n = 109$ and $n = 108$ we have

$$h\nu = \frac{13.6}{108^2} - \frac{13.6}{109^2}$$

or

$$\nu = 5.15 \times 10^9 \text{ Hz}$$

or

$$\lambda = c/\nu = 5.83 \text{ cm}$$

(b) For such a highly excited states, atoms are easily ionized by collididing with other atoms

At the same time, the probability of a transition between the highly excited states is very small. It is very difficult to produce such environment in a laboratory in which the probability of a collision is very small and yet there are sufficiently many such highly excited atoms available.

More recently the availability of powerful lasers may make it possible to stimulate an atom to such a highly excited states by multiphoton excitation.

b) For such highly excited states the effective nuclear charge of the helium atom experienced by an orbital electron is approximately equal to that of a photon. Hence for such ~~tr~~ transitions the wavelength from He approximately equals that of H.

4. Sodium atom

(a) $\mathcal{E}_{iz} = 5.14 \text{ eV} = -\mathcal{E}_3$

For the 3s outer electron of sodium

$$Z_{eff} = n\sqrt{\frac{\mathcal{E}_n}{-13.6\text{eV}}}$$

$$= 3\sqrt{\frac{-5.14}{-13.6}} = 1.84$$

The simple screening model predicts $Z_{eff} = 1$, so clearly the 3s electron is slightly penetrating the inner orbits an so is less screened by the inner electrons.

(b)

For the 4f state

$$Z_{eff} = n\sqrt{\frac{-E_n}{-13.6\,eV}} = 4\sqrt{\frac{-0.85}{-13.6}} = 1.00$$

so the screening is complete, with the 11 positive charges in the nucleus screened by the 10 electrons in the n=1 and n=2 shells.

4. Jónunarorka litíns

(a) Ef við gerum ráð fyrir að kjarnhleðslan sé skermuð með 1s rafeindum, þá erum við með vetnis-atóm með jónunarorku $\varepsilon_{iz} = 13.6\,eV$. Þetta er mun stærra er uppgefið gildi.

Nú er gildisrafeindin 2s en ekki 1s eins og í vetnisatómi. Því er jónunarorkan mun lægri eða

$$\varepsilon_{iz} = \frac{R_\infty}{n^2} = \frac{13.6\ eV}{4} = 3.4\ eV$$

sem er lægra en upp-
gefið gildi.

Nú vitum við að 2s
rafeind smýgur um kjarnann
og þar með er skermun
vegna 1s rafeinda ekki
svo öflug. Þetta eykur
aðdráttarkraftinn þ.a.
bindiorkan eykst frá
3.4 eV upp í 5.39 eV.

(b) 3ja jónunarorka Li

Bindiorka n-ástands er gefin með

$$E_n = -\frac{Z^2}{n^2} R_y$$

Með $Z = 3$ og $n = 1$, er 3ja jónunarorkan

$$\mathcal{E}_{i3} = \frac{9 \times 13.6\,eV}{1^2} = 122.4\,eV$$

Þetta gildi er nákvæmt þar sem engar aðrar rafeindir eru til staðar.

2. Hydrogen atom

The preturbation caused by the finite volume of the proton is

$$H' = \begin{cases} 0 & r \geq R \\ \dfrac{e^2}{r} - \dfrac{e^2}{R}\left(\dfrac{3}{2} - \dfrac{r^2}{2R^2}\right), & r < R \end{cases}$$

The unperturbed wave function is

$$\psi = R(r)\,\Theta\,\Phi = R(r)\,Y_{00}$$

$$= \frac{1}{2^{3/2} a_0^{3/2}} e^{-r/a_0 2}\left(2 - \frac{r}{a_0}\right) Y_{00}$$

and making the approximation that $r << a_0$ so

$$\exp(-r/2a_0) \simeq 1$$

and

$$\left(2 - \frac{r}{a_0}\right) \simeq 2$$

we have

$$\psi = \frac{2}{(\cancel{n}a_0)^{3/2}} Y_{00} = \frac{2}{(\cancel{n}a_0)^{3/2}} \cdot \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{2\pi}}$$

and

$$\Delta E_{ns} = \langle \psi_{ns}^{*} | H' | \psi_{ns} \rangle$$

$$= \int_0^R \left[\frac{e^2}{r} - \frac{e^2}{R}\left(\frac{3}{2} - \frac{r^2}{2R^2}\right)\right] \psi_{ns}^{*} \psi_{ns} r^2 dr d\Omega$$

$$= \frac{2\pi}{5} \frac{e^2 R^2}{\pi (na_0)^3}$$

and since

$$\psi_{ns}(0) = \frac{1}{\sqrt{\pi}\,(n a_0)^{3/2}}$$

or for $n=2$,

$$\psi_{2s}(0) = \frac{1}{\sqrt{\pi}\,(2a_0)^{3/2}}$$

we have

$$\Delta E_{ns} = \frac{2\pi}{5} e^2 |\psi_{ns}(0)|^2 R^2$$

As non-s wave functions have much smaller fraction inside the nucleus an so cause smaller perturbation, the energy shift is much smaller.

For hydrogen atom since

$$\Delta E_{2p} \ll \Delta E_{2s}$$

we have

$$\Delta E_{ps} = \Delta E_{2s} - \Delta E_{2p} \approx \Delta E_{2s}$$

$$= \frac{2\pi}{5} e^2 |\psi_{2s}(0)|^2 R$$

where

$$\psi_{2s}(0) = (2a_0)^{-3/2} \pi^{-1/2}$$

Hence

$$\Delta E_{ps} \approx \frac{2\pi}{5} e^2 \left[(2a_0)^{-3/2} \pi^{-1/2}\right]^2 R^2$$

$$= \frac{e^2 R^2}{20 a_0^3} = \left(\frac{e^2}{\hbar c}\right)^2 \frac{R^2 mc^2}{20 a_0^2}$$

$$= \left(\frac{1}{137}\right)^2 \times \frac{10^{-26} \times 0.511 \times 10^6}{20 \times (5 \times 10^{-9})^2}$$

$$\simeq 5.4 \times 10^{-10} \text{ eV}$$