Háskóli Íslands | Haust 2014

Verkfræði- og náttúruvísindasvið

Raunvísindadeild

Eðlisfræði

EÐL523M Framleiðsla smárása

Lokapróf

8. desember 2014, kl. 09:00 - 12:00

1. (16) **Kennistærðir MOSFET – MOSFET characteristics**

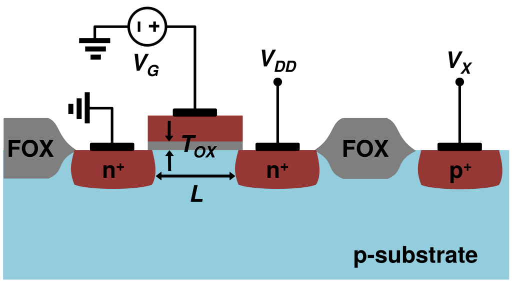
   Fyrir þversniðið af NMOS tólinu sem er sýnt hér að neðan, gerum við ráð fyrir rásalengd $L = 0.5$ μm, breidd rásar $W = 50$ μm, þykkt gáttaroxíðs $t_{\rm ox} = 9$ nm, Fermiorkustigi $\psi_{\rm b} = 0.4$ V, þröskuldsspennu við enga álagða spennu $V_{\rm Th0} = 0.7$ V, íbótarþéttleika undirlags 9×10^{14} cm^{-3}, og hlutfallslegum rafsvörunarstuðli Si sem 11.8 og fyrir SiO_2 3.9 og hreyfanleika $\mu = 350$ cm^2/Vs.

   (a) Reiknaþröskuldsspennuna þegar $V_{\rm X} = 0.5$ V, $V_{\rm DD} = 5$ V, og $V_{\rm G} = 2$ V.

   (b) Haldið áfram með (a) og reikniðs svelgstrauminn $I_{\rm D}$.

   (c) Endurtakið (b) með $V_{\rm X} = 0$ V.

   (d) Gerum ráð fyrir að $V_{\rm X}$ megi breyta á meðan öðrum spennugildum er haldið föstum. Reikna við hvaða spennu $V_{\rm X}$ veldur því að tólið er cut off.



Consider the cross-section of an NMOS device shown below. Assume the channel length $L = 0.5\ \mu$m, the channel width $W = 50\ \mu$m, the gate oxide thickness $t_{\mathrm{ox}} = 9$ nm, the Fermi level $\psi_{\mathrm{b}} = 0.4$ V, the zero-bias threshold voltage $V_{\mathrm{Th0}} = 0.7$ V, the substrate doping concentration 9×10^{14} cm^{-3}, and the relative permittivity of Si is 11.8 and of SiO_2 3.9, respectively. Assume a mobility of $\mu = 350$ cm^2/Vs.

(a) Calculate the threshold voltage when $V_{\mathrm{X}} = 0.5$ V, $V_{\mathrm{DD}} = 5$ V, and $V_{\mathrm{G}} = 2$ V.

(b) Continuing from (a) calculate the drain current I_{D}.

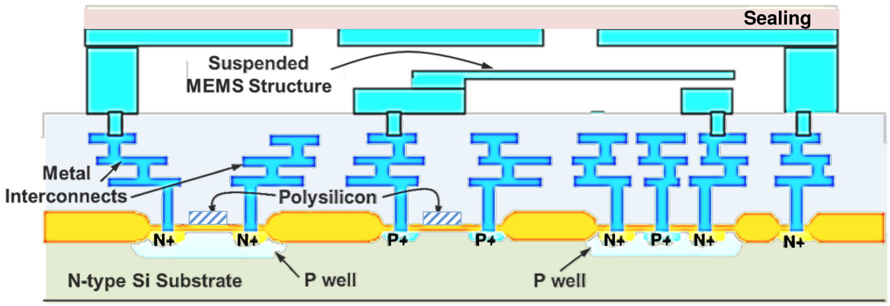
(c) Repeat (b) with $V_{\mathrm{X}} = 0$ V.

(d) Now, suppose V_{X} can be varied while other voltage biases are fixed. Calculate the value of V_{X} that causes the device to cut off.

2. (8) **Device Cross-Sections/Symbols**

Skoðið þversniðið af MEMS-smára smárásinni hér að neðan. Merkið smárana (þ.e. tveir MOS og einn tvískeyttur smári) og teiknið inn á myndina tákn fyrir sérhvern smára á viðeigandi stöðum., þ.e. örin í tákninu yfir rétta svæðið o.s.frv.

Consider the merged MEMS-transistor integrated circuit cross-section shown below. Identify the transistors (i.e., two MOS and one bipolar transistor) and draw the corresponding symbols for each of the transistors on top of the figure in the appropriate positions, i.e., with the "arrow part" of the symbol over the right region, etc.



3. (6) **Kísill – Silicon**

   Af hverju er kísill mikilvægasta frumefnið á smárásum ? Nefnið þrjú atriði.

   Why is silicon the most important element in integrated circuits ? Name three reasons.

4. (15) **CVD**

Gerum ráð fyrir að $SiCl_4$ lind sé notuð til að rækta fjölkristallaðan kísil með efnagufuágræðslu (CVD). Gerum ráð fyrir að þéttleiki $SiCl_4$ concentration í gas flæðinu N_g sé 5×10^{16} molecules/cm^3, massaflutningsstuðullinn h_g sé 0.63 cm/sec, hraðafasti fyrir yfirborðshvörf k_s sé $2 \times 10^6 \exp(-1.9/kT)$ cm/sec, og atómþéttleiki fjölkristallaðs kísils sé 5×10^{22} atoms/cm^3.

(a) Áætlið ræktunarhraða fjölkristallaðs kísils, gerið ráð fyrir massaflutningur takmarki CVD ferlið.

(b) Við hvaða hitastig er massaflutningssstuðullinn h_g jafn hraðafasta fyrir yfirborðshvörf k_s ? Hver er ræktunarhraðinn við þetta hitastig ?

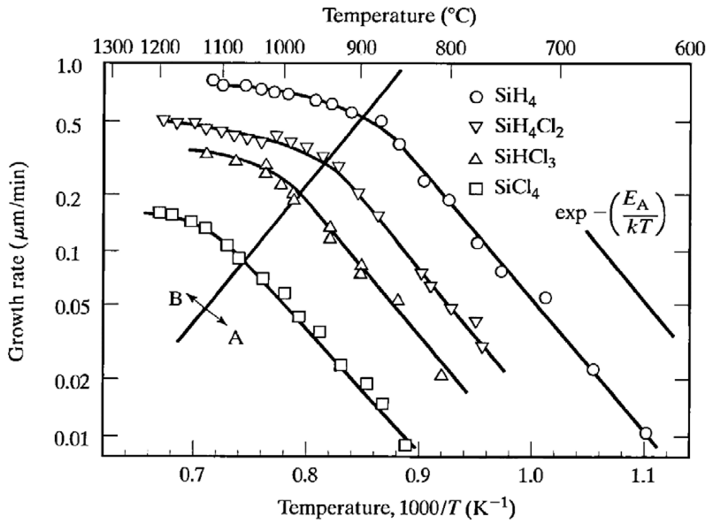
(c) Myndin hér að neðan sýnir ræktunarhraða fyrir fjölkristallaðann kísil frá fjórum mismunandi gaslindum. Út frá ferlinum fyrir SiH_4, metið örvunarorkuna E_A (í eV) fyrir hraðafasta yfirborðshvarfsins.

Suppose a $SiCl_4$ source is used to deposit polysilicon via chemical vapor deposition (CVD). Assume the $SiCl_4$ concentration in the gas stream N_g is 5×10^{16} molecules/cm^3, the mass-transfer coefficient h_g is 0.63 cm/sec, the surface-reaction rate constant k_s is $2 \times 10^6 \exp(-1.9/kT)$ cm/sec, and the polysilicon atom density is 5×10^{22} atoms/cm^3.

(a) Estimate the polysilicon film growth rate, assuming the CVD process is mass-transfer limited.

(b) At what temperature does the mass-transfer coefficient h_g equal the surface-reaction rate constant k_s ? What is the growth rate at this temperature ?

(c) The following figure shows the deposition rates of polysilicon for four different gas sources. From the SiH_4 curve, estimate the activation energy E_A (in eV) for the surface-reaction rate constant.


Temperature (°C)
1300 1200 1100 1000 900 800 700 600
1.0
0.5
0.2
0.1
0.05
0.02
0.01
Growth rate (μm/min)
SiH4
SiH4Cl2
SiHCl3
SiCl4
exp −(EA/kT)
B
A
0.7
0.8
0.9
1.0
1.1
Temperature, 1000/T (K−1)

5. (8) **Valvísi í ætingu – Selectvity in etching**

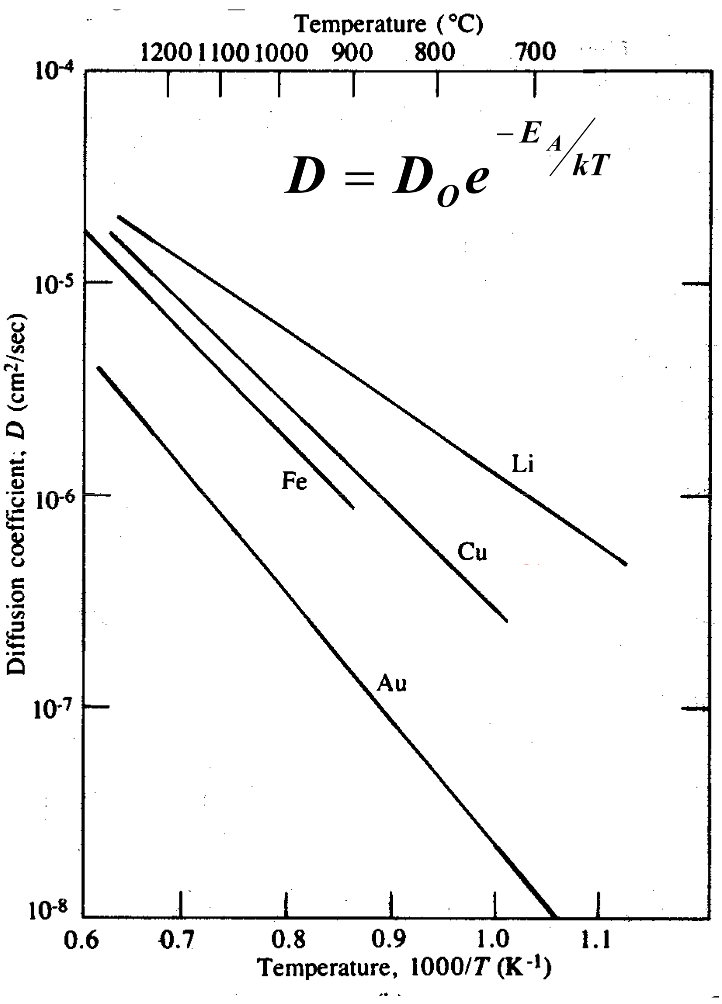
Finna skal hvaða valvísi skal krafist þægar ætt er 460 nm laga af fjölkristölluðum kísli án þess að æta meira en 2 nm niður í undirliggjandi gáttaroxíð. Gera skal ráð fyrir að einsleitni ætingar á fjölkristölluðum kísli sé 10 %.

Find the etch selectivity required to etch a 460 nm polysilicon layer without removing more than 2 nm of its underlying gate oxide, assuming that the polysilicon is etched with a process having a 10 % etch rate uniformity.

6. (15) **Sveim – Diffusion**

Gulli er sveimað inn í kísilskífu með föstum yfirborðsþéttleika 10^{18} cm^{-3}. Hve lengi er gullið að sveima fullkomlega um kíslskífu sem er 400 μm þykk þegar bakgrunnsþéttleiki er 10^{16} cm^{-3} við hitastig 1000°C ?

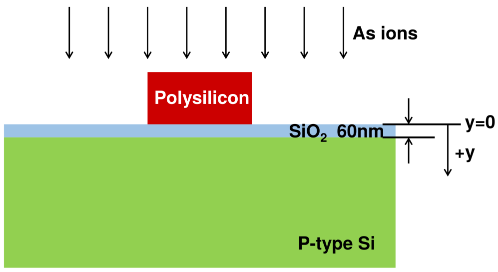
Gold is diffused into a silicon wafer using a constant-source diffusion with a surface concentration of 10^{18} cm^{-3}. How long does it take the gold to diffuse completely through a silicon wafer 400 μm thick with a background concentration of 10^{16} cm^{-3} at a temperature of 1000°C ?



7. (20) **Jónaígræðsla – Ion implantation**

Gerum ráð fyrir eftirfarandi þversnisði sem skal íbætt með As jónaígræðslu til að mynda lindar/svelg svæðin. Gerum ráð fyrir að Si undirlagið sé upphaflega íbætt með B með einsleitum íbótarþéttleika 10^{16} cm^{-3}.

Consider the following cross-section that is to be doped with As using ion implantation to form the source/drain regions. Assume the Si substrate is initially doped with B with a uniform concentration of 10^{16} cm^{-3}.



(a) Gera skal ráð fyrir að SiO_2 og fjölkristallaður kísill hafi sama stöðvunarafl og Si, og að þykktin á SiO_2 sé 60 nm. Hver er jónaígræðsluskammturinn og orkan sem þarf til að fá hámarksþéttleika 10^{19} cm^{-3} fyrir As við samskeyti SiO_2 og Si við lindar/svelg svæðin (þ.e., $y = 60$ nm) ?

(b) Halda áfram frá (a), og reikna dýpt samskeytanna á lindar/svelg svæðunum.

(c) Hver er minnsta þykkt á fjölkristallaða kíslinum ef fjölkristallaði kísillinn og SiO_2 hlaðinn á að þjóna sem virk gríma fyrir jónaígræðsluna sem minnkar As þéttleikann í rásasvæðinu niður fyrir 1/10 af bakgrunnsíbótinni ?

(d) Ef haldið er áfram frá (a), eftirfylgjandi drif skref við 1100°C gefur endanlega skeytadýpt sem er 2 μm (mælt frá SiO_2/Si samskeytunum). Áætlið endanlegt sheet viðnám í S/D svæðunum.

(e) Halda áfram frá (d), áætlið hve langur drif tíminn er.

(a) Assume that the SiO_2 and polysilicon layers have the same ion stopping power as Si, and that SiO_2 thickness is 60 nm. What are the ion implantation dose and energy required to achieve a peak concentration of 10^{19} cm^{-3} of As at the SiO_2 and Si interface in the source/drain regions (i.e., $y = 60$ nm) ?

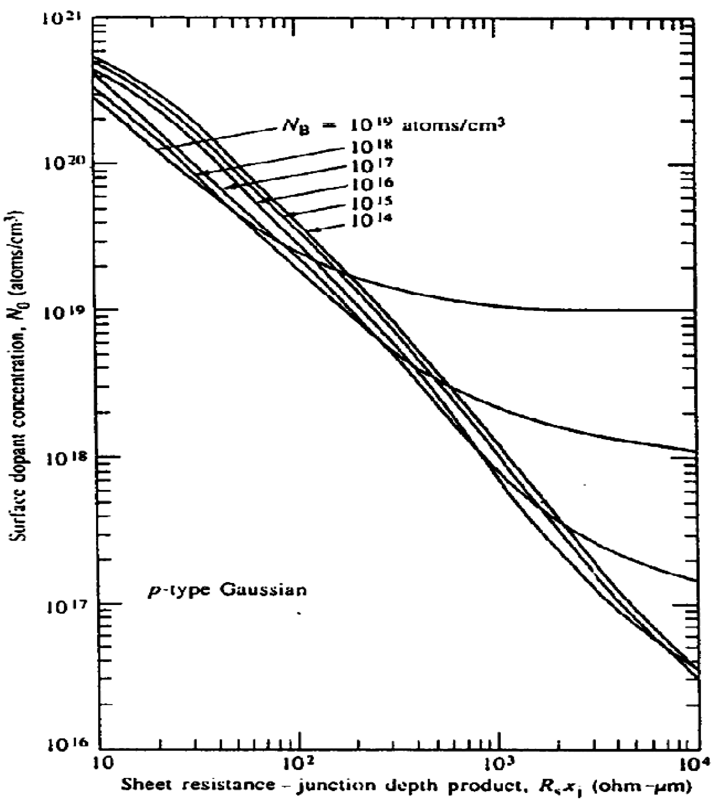
(b) Continuing from (a), calculate the junction depth of the source/drain regions.

(c) What is the minimal thickness of the gate polysilicon for the polysilicon and SiO_2 stack to serve as an effective implantation mask that decreases the As concentration in the channel region below 1/10th the background concentration ?

(d) Continuing from (a), a following drive-in step at 1100°C yields a final junction depth of 2 μm (counted from the SiO_2 and Si interface). Estimate the final sheet resistance in the S/D regions.

(e) Continuing from (d), estimate the required drive-in time.

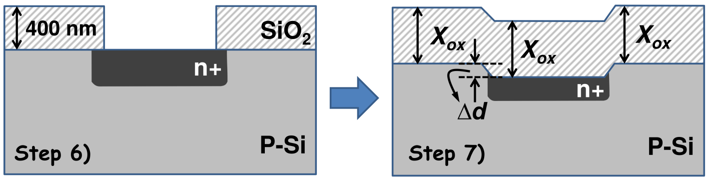
	Si	B	In	As	Sb	P	Units
D^0	560	1.0	1.2	9.17	4.58	4.70	$cm^2\ sec^{-1}$
E_A	4.76	3.5	3.5	3.99	3.88	3.68	eV



8. (12) **Oxun – Oxidation**

Gerum ráð fyrir að atómþéttleiki í hreinum kísli sé 5×10^{22} cm^{-3} og að sameindaþéttleiki SiO_2 sé 2.2×10^{22} cm^{-3}. Strúktúrinn hér að neðan hefur farið í gengum eftirfarandi frmaleiðsluskref:

Assume the atom density of pure Si is 5×10^{22} cm^{-3} and the molecular density of SiO_2 is 2.2×10^{22} cm^{-3}. The structure shown below has gone through the following process steps:

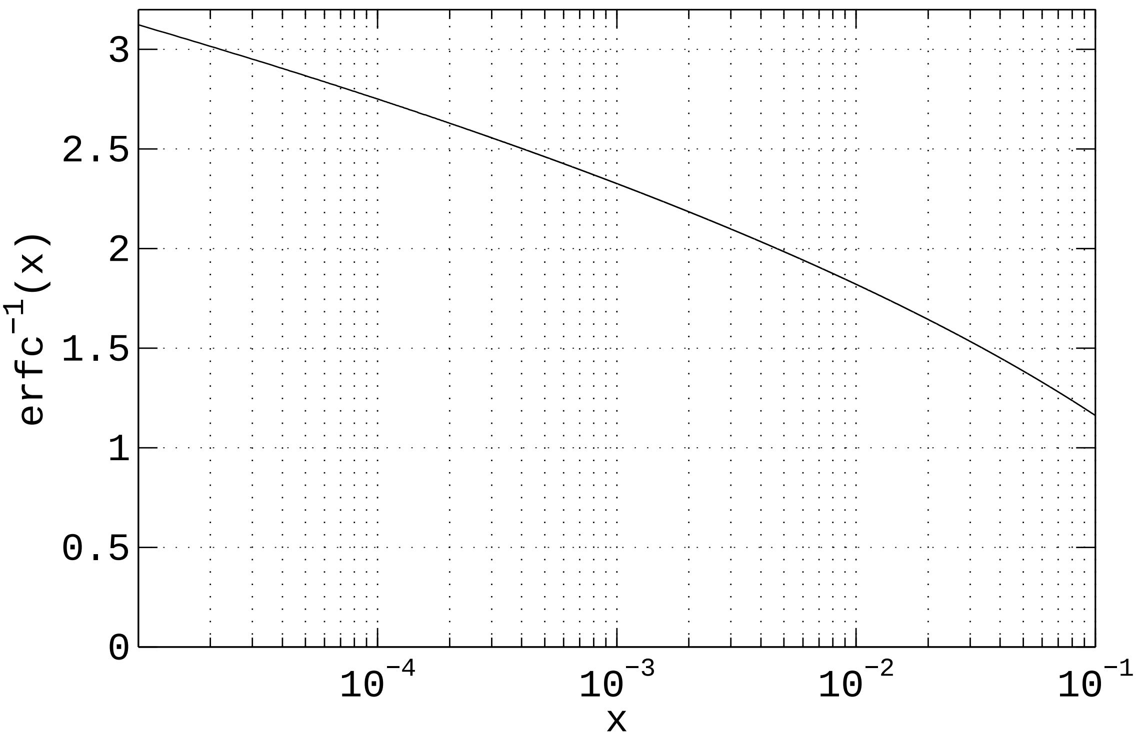


1) Start with a bare <100> silicon wafer
2) Grow 400 nm of oxide
3) Lithography
4) Dry etch 400 nm of oxide
5) Remove P.R.
6) Diffuse phosphorous (n+)
7) Wet oxidation for t minutes

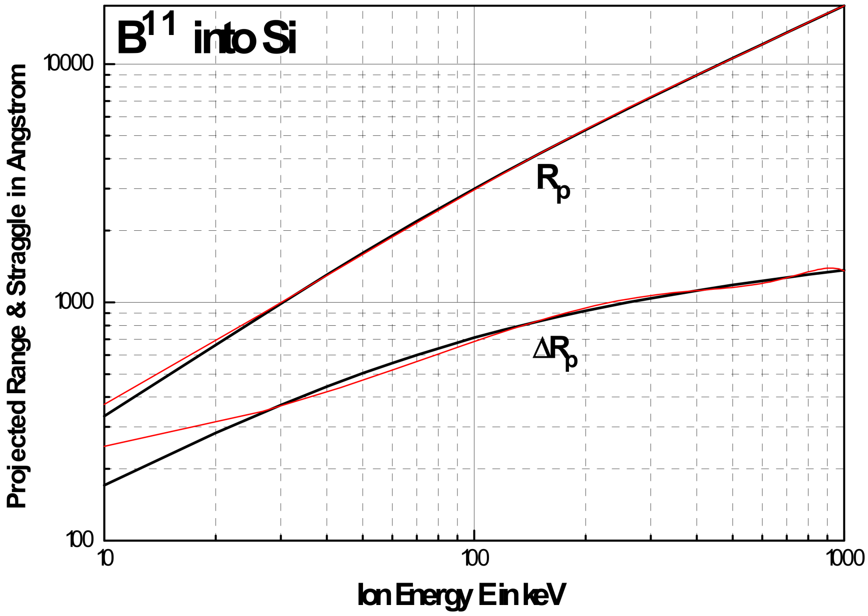
Gerum ráð fyrir að línulegur hraðafasti oxunar sé $(B/A)_{n^+} = 4 \times (B/A)_p = 0.4$ μm/klst (þ.e., yfirborðshraðafastinn eykst þegar fosfór er fyrir hendi), og fleygboga hraðafastar eru $(B)_{n^+} = (B)_p = 0.2$ μm^2/klst. Ákvarða t, Δd, og X_{ox}.

Assume that the linear oxidation rate constants are $(B/A)_{n^+} = 4 \times (B/A)_p = 0.4$ μm/hr (i.e., the surface reaction rate increases when phosphorous is present), and the parabolic constants are $(B)_{n^+} = (B)_p = 0.2$ μm^2/hr. Determine t, Δd, and X_{ox}.

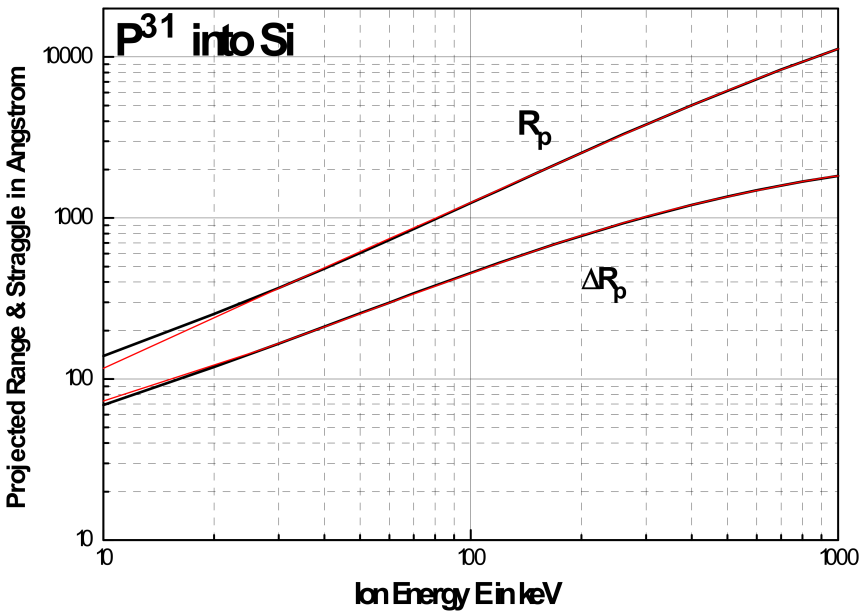
Gagnleg gröf

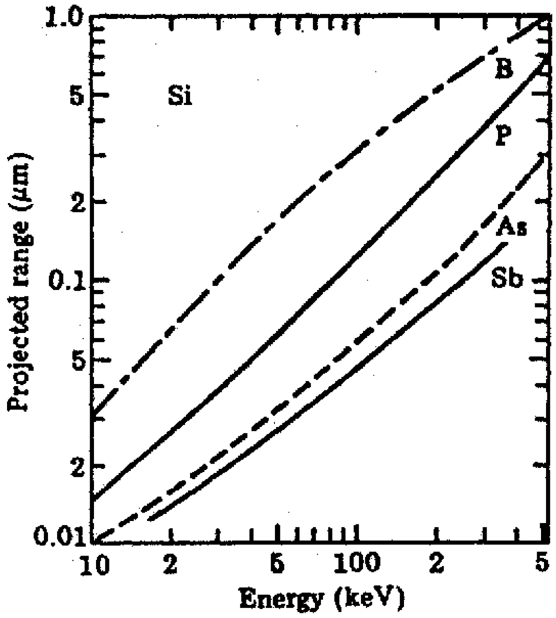


$$R_p = 51.051 + 32.60883\,E - 0.03837\,E^2 + 3.758e\text{-}5\,E^3 - 1.433e\text{-}8\,E^4$$
$$\Delta R_p = 185.34201 + 6.5308\,E - 0.01745\,E^2 + 2.098e\text{-}5\,E^3 - 8.884e\text{-}9\,E^4$$

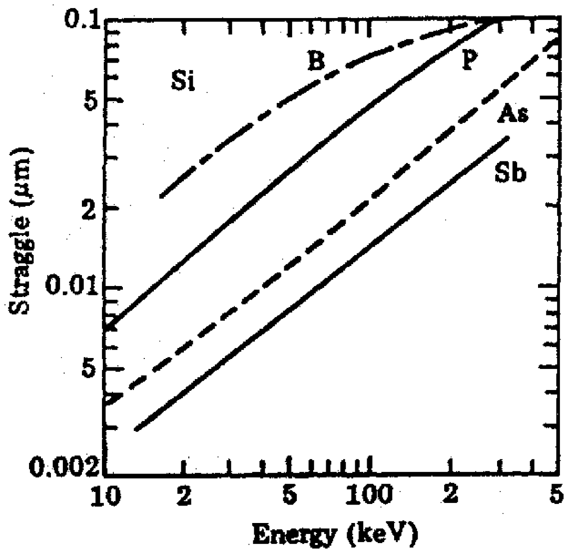


$$R_p = -7.14745 + 12.33417\,E + 0.00323\,E^2 - 8.086e\text{-}6\,E^3 + 3.766e\text{-}9\,E^4$$
$$\Delta R_p = 24.39576 + 4.93641\,E - 0.00697\,E^2 + 5.858e\text{-}6\,E^3 - 2.024e\text{-}9\,E^4$$





1.0
5
2
0.1
5
2
0.01
10
2
5
100
2
5
Si
B
P
As
Sb
Projected range (μm)
Energy (keV)



0.1
5
2
0.01
5
0.002
10
2
5
100
2
5
Si
B
P
As
Sb
Straggle (μm)
Energy (keV)

1 Fastar

$$q = 1.602 \times 10^{-19} \quad \text{C}$$

$$N_{\text{Av}} = 6.022 \times 10^{23} \quad \text{sameindir/mól}$$

$$k = 1.38 \times 10^{-23} \text{J/K} = 8.62 \times 10^{-5} \quad \text{eV/K}$$

$$\epsilon_0 = 8.854 \times 10^{-14} \quad \text{F/cm}$$

$$\epsilon_{\text{ox}}/\epsilon_0 = 3.9$$

$$\epsilon_{\text{Si}}/\epsilon_0 = 11.9$$

$$\epsilon_{\text{Ge}}/\epsilon_0 = 16$$

$$\epsilon_{\text{GaAs}}/\epsilon_0 = 13.1$$

Fyrir kísil við stofuhita:

$$n_{\text{i}} = 9.65 \times 10^9 \quad \text{cm}^{-3}$$

Fyrir GaAs við stofuhita:

$$n_{\text{i}} = 2.25 \times 10^9 \quad \text{cm}^{-3}$$

2 Ræktun

$$k_0 = \frac{C_{\text{s}}}{C_{\text{l}}}$$

$$C_{\text{s}} = k_0 C_0 \left[1 - \frac{M}{M_0}\right]^{k_0 - 1}$$

3 Hálfleiðarar

$$E_{\text{H}} = -\frac{m_{\text{e}} q^4}{8\epsilon_0^2 h^2 n^2} = -\frac{13.6}{n^2}$$

$$E_{\text{g}} = 1.17 - \frac{(4.73 \times 10^{-4}) T^2}{(T + 636)} \qquad \text{kísill}$$

$$E_{\text{g}} = 1.52 - \frac{(5.4 \times 10^{-4}) T^2}{(T + 204)} \qquad \text{GaAs}$$

$$m^* = \frac{\hbar^2}{d^2 E / dk^2}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_{\text{F}}}{kT}\right)}$$

$$n = \int_{\infty}^{E_{\text{c}}} f(E) N(E) dE$$

$$N(E) = 4\pi \left(\frac{2m^*}{h^2}\right)^{3/2} E^{1/2}$$

$$f(E) \approx \exp\left(-\frac{E - E_{\text{F}}}{kT}\right) \text{ ef } E - E_{\text{F}} > 3kT$$

$$f(E) \approx 1 - \exp\left(-\frac{E_{\text{F}} - E}{kT}\right) \text{ ef } E - E_{\text{F}} < 3kT$$

$$n \approx N_{\text{c}} \exp\left(-\frac{E_{\text{c}} - E_{\text{F}}}{kT}\right)$$

$$N_{\text{c}} = 2 \left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$p \approx N_{\text{v}} \exp\left(-\frac{E_{\text{F}} - E_{\text{v}}}{kT}\right)$$

$$N_{\text{v}} = 2 \left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$np = N_c N_v \exp\left(-\frac{E_g}{kT}\right) = n_i^2$$

$$n = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$$

$$p = n_i \exp\left(\frac{E_i - E_F}{kT}\right)$$

$$E_c - E_F = kT \ln\left(\frac{N_c}{N_D}\right)$$

$$E_F - E_v = kT \ln\left(\frac{N_v}{N_A}\right)$$

$$np = n_i^2$$

Við stofuhita fyrir kísil

$$N_c = 2.8 \times 10^{19} \quad \text{cm}^{-3}$$

$$N_v = 1.04 \times 10^{19} \quad \text{cm}^{-3}$$

Við stofuhita fyrir GaAs

$$N_c = 4.7 \times 10^{17} \quad \text{cm}^{-3}$$

$$N_v = 7 \times 10^{18} \quad \text{cm}^{-3}$$

n-leiðandi hálfleiðari

$$n_n = \frac{1}{2}\left[N_D - N_A + \sqrt{(N_D - N_A)^2 + 4n_i^2}\right]$$

og

$$p_n = \frac{n_i^2}{n_n}$$

p-leiðandi hálfleiðari

$$p_p = \frac{1}{2}\left[N_A - N_D + \sqrt{(N_A - N_D)^2 + 4n_i^2}\right]$$

og

$$n_p = \frac{n_i^2}{p_p}$$

$$N_C = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$N_V = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$J = \sigma\mathcal{E}$$

$$\sigma = \frac{nq^2\tau}{m_n^*} \qquad [\Omega\text{cm}]^{-1}$$

$$\sigma = qn\mu_n$$

$$\mu_n = \frac{q\tau}{m_n^*}$$

$$J = q(n\mu_n + p\mu_p)\mathcal{E} = \sigma\mathcal{E}$$

$$R = \frac{\rho L}{Wd} = \frac{L}{Wd}\frac{1}{\sigma}$$

4 MOSFET

MOS kjörtvistur

$$q\phi_{ms} = q(\phi_m - \phi_s)$$

$$q\phi_{ms} = q\phi_m - \left[q\chi + \frac{E_g}{2} + q\phi_b\right]$$

$$Q_{sc} = -qN_A x_{dmax} \approx -\sqrt{2q\epsilon_s N_A (2\psi_b)}$$

$$\psi_s(\text{umhverfing}) \approx 2\psi_b = \frac{2kT}{q} \ln\left(\frac{N_A}{n_i}\right)$$

$$x_{\mathrm{dmax}} = \left(\frac{2\epsilon_{\mathrm{s}}\psi_{\mathrm{s}}(\mathrm{umhv.})}{qN_{\mathrm{A}}}\right)^{1/2} \approx \left(\frac{2\epsilon_{\mathrm{s}}(2\psi_{\mathrm{b}})}{qN_{\mathrm{A}}}\right)^{1/2}$$

$$C_{\mathrm{ox}} = \frac{\epsilon_{\mathrm{ox}}}{d}$$

$$C_{\mathrm{d}} = \frac{\epsilon_{\mathrm{s}}}{x_{\mathrm{d}}}$$

$$C = \frac{C_{\mathrm{ox}}C_{\mathrm{d}}}{C_{\mathrm{ox}} + C_{\mathrm{d}}}$$

$$I_{\mathrm{D}} \approx \frac{W}{L}\mu_{\mathrm{n}}C_{\mathrm{ox}}(V_{\mathrm{G}} - V_{\mathrm{T}})V_{\mathrm{D}}$$

ef $V_{\mathrm{D}} \ll (V_{\mathrm{G}} - V_{\mathrm{T}})$

$$V_{\mathrm{T}} \approx \frac{\sqrt{2\epsilon_{\mathrm{s}}qN_{\mathrm{A}}(2\psi_{\mathrm{b}})}}{C_{\mathrm{ox}}} + 2\psi_{\mathrm{b}}$$

$$V_{\mathrm{Dsat}} \approx V_{\mathrm{G}} - 2\psi_{\mathrm{b}} + K^2\left[1 - \left(1 + \frac{2V_{\mathrm{G}}}{K^2}\right)^{1/2}\right]$$

þar sem

$$K \equiv \frac{\epsilon_{\mathrm{Si}}qN_{\mathrm{A}}}{C_{\mathrm{ox}}}$$

$$I_{\mathrm{Dsat}} \approx \frac{W\mu_{\mathrm{n}}C_{\mathrm{ox}}}{2L}(V_{\mathrm{G}} - V_{\mathrm{T}})^2$$

Frávik frá kjörhegðan

$$V_{\mathrm{FB}} = \phi_{\mathrm{ms}} - \frac{Q_{\mathrm{it}}}{C_{\mathrm{ox}}}$$

$$V_{\mathrm{T}} = \phi_{\mathrm{ms}} - \frac{Q_{\mathrm{it}}}{C_{\mathrm{ox}}} - \frac{Q_{\mathrm{sc}}}{C_{\mathrm{ox}}} + 2\psi_{\mathrm{b}}$$

$$\psi_{\mathrm{b}} = (E_{\mathrm{i}} - E_{\mathrm{F}})/q$$

$$\Delta V_{\mathrm{T}} = \frac{\sqrt{2q\epsilon_{\mathrm{s}}N_{\mathrm{A}}}}{C_{\mathrm{o}}}\left[(2\psi_{\mathrm{b}} + V_{\mathrm{BS}})^{1/2} - (2\psi_{\mathrm{b}})^{1/2}\right]$$

$$x_{\mathrm{S}} = \left(\frac{2\epsilon_{\mathrm{s}}}{qN_{\mathrm{A}}}(V_{\mathrm{bi}} + V_{\mathrm{BS}})\right)^{1/2}$$

$$x_{\mathrm{D}} = \left(\frac{2\epsilon_{\mathrm{s}}}{qN_{\mathrm{A}}}(V_{\mathrm{D}} + V_{\mathrm{bi}} + V_{\mathrm{BS}})\right)^{1/2}$$

5 Viðnám

$$R = \frac{\rho L}{A}$$

$$\sigma = \frac{1}{\rho} = (q\mu_{\mathrm{n}}n + q\mu_{\mathrm{p}}p)$$

$$R = \frac{1}{G} = \frac{L}{W}\frac{1}{g}$$

6 Hreyfifræði gass

$$pV = RT = N_{\mathrm{Av}}kT$$

$$f_{\mathrm{v}} = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$

$$\phi = \frac{p}{(2\pi mkT)^{1/2}} = 3.51 \times 10^{22}\left(\frac{p}{\sqrt{MT}}\right)$$

7 Lagvöxtur

$$C_{\mathrm{s}} = \frac{C_{\mathrm{g}}}{1 + (k_{\mathrm{s}}/h_{\mathrm{g}})}$$

$$v = \frac{k_{\mathrm{s}}h_{\mathrm{g}}}{k_{\mathrm{s}} + h_{\mathrm{g}}}\left(\frac{C_{\mathrm{t}}}{C_{\mathrm{a}}}\right)y$$

$$\delta(x) \approx \sqrt{\frac{\mu x}{\rho_{\mathrm{d}}v}}$$

$$\bar{\delta}(x) = \frac{1}{L}\int_0^L \delta(x)dx = \frac{2}{3}\sqrt{\frac{\mu L}{\rho_{\mathrm{d}}v}}$$

$$h_{\mathrm{g}} = \frac{D_{\mathrm{g}}}{\delta} = \frac{3}{2}D_{\mathrm{g}}\sqrt{\frac{v\rho_{\mathrm{d}}}{\mu L}}$$

8 Oxun

$$x^2 + \frac{2D}{k}x = \frac{2DC_0}{C_1}(t+\tau)$$

$$\tau \equiv \left(d_0^2 + \frac{2Dd_0}{k}\right)\frac{C_1}{2DC_0}$$

$$x = \frac{D}{k}\left[\left(1 + \frac{2C_0k^2(t+\tau)}{DC_1}\right)^{1/2} - 1\right]$$

$$x^2 + Ax = B(t+\tau)$$

$$A \equiv \frac{2D}{k}$$

$$B \equiv \frac{2DC_0}{C_1}$$

$$\frac{B}{A} \equiv \frac{kC_0}{C_1}$$

9 Sveim

$$F = -D\frac{\partial C}{\partial x}$$

$$\frac{\partial C}{\partial t} = -\frac{\partial F}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right)$$

$$D = D_0 \exp\left(-\frac{E_\mathrm{a}}{kT}\right)$$

Fastur yfirborðsþéttleiki

$$C(x,t) = C_\mathrm{s}\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$Q(t) = \int_0^\infty C(x,t)dx$$

$$Q(t) = \frac{2}{\sqrt{\pi}}C_\mathrm{s}\sqrt{Dt} \approx 1.13C_\mathrm{s}\sqrt{Dt}$$

Föst heildaríbót

$$\int_0^\infty C(x,t)dx = S$$

$$C(\infty,t) = 0$$

$$C(x,t) = \frac{S}{\sqrt{\pi Dt}}\exp\left(-\frac{x^2}{4Dt}\right)$$

$$C_\mathrm{s}(t) = \frac{S}{\sqrt{\pi Dt}}$$

10 Jónaígræðsla

$$\left(\frac{dE}{dx}\right)_\mathrm{tot} = S_\mathrm{n}(E) + S_\mathrm{e}(E)$$

$$\int_0^R dx = R = -\int_E^0 \frac{dE}{(dE/dx)_\mathrm{tot}}$$

$$S_\mathrm{e} = k_\mathrm{e}\sqrt{E}$$

$$N(x) = N_\mathrm{p}\exp\left[-\frac{(x-R_\mathrm{p})^2}{2(\Delta R_\mathrm{p})^2}\right]$$

$$Q = \int_0^\infty N(x)dx$$

$$Q = \sqrt{2\pi}N_\mathrm{p}\Delta R_\mathrm{p}$$

$$N(x) = \frac{N_\mathrm{p}}{\left[1 + \frac{4Dt}{2(\Delta R_\mathrm{p})^2}\right]^{1/2}}\exp\left[-\frac{(x-R_\mathrm{p})^2}{2(\Delta R_\mathrm{p})^2 + 4Dt}\right]$$

11 Málmar

$$\text{MTF} \sim \frac{1}{J^2} \exp\left(\frac{E_\text{a}}{kT}\right)$$

12 Lithography

$$\text{CD} = W_\text{min} \approx \sqrt{\lambda g}$$

$$R = \frac{0.61\lambda}{n \sin\theta}$$

$$R = \frac{0.61\lambda}{\text{NA}} = k_1 \frac{\lambda}{\text{NA}}$$

$$\text{NA} \equiv n \sin\theta$$

$$\text{DOF} = \pm\frac{R/2}{\tan\theta} \approx \pm\frac{R/2}{\sin\theta} = \pm k_2 \frac{\lambda}{(\text{NA})^2}$$

$$\text{MTF} = \frac{I_\text{MAX} - I_\text{MIN}}{I_\text{MAX} + I_\text{MIN}}$$

$$\text{CMTF}_\text{viðnám} = \frac{E_\text{T} - E_1}{E_\text{T} + E_1} = \frac{10^{1/\gamma} - 1}{10^{1/\gamma} + 1}$$

$$\gamma = \frac{1}{\log\left[\frac{E_\text{T}}{E_1}\right]}$$

13 Framleiðni

Líkan Poisson

$$Y = \frac{1}{\text{e}^{AD}}$$

Líkan Murphy

$$Y = \left[\frac{1 - \text{e}^{-AD}}{AD}\right]^2$$

Líkan Seeds

$$Y = \frac{1}{\text{e}^{\sqrt{AD}}}$$