Háskóli Íslands Haust 2016

Verkfræði- og náttúruvísindasvið

Raunvísindadeild

Eðlisfræði

EÐL523M Framleiðsla smárása

Lokapróf

13. desember 2016, kl. 09:00 - 12:00

1. (24) **Íbæting – Doping**

Í jónaígræðsluskrefi (skammtur = 10^{14} cm^{-2}, $R_\mathrm{p} = 0.0226\,\mu$m, $\Delta R_\mathrm{p} = 0.0102\,\mu$m) er arsen (As) ígrætt í p-leiðandi hálfleiðara sem hefur einsleita bór (B) íbót af þéttleika 10^{16} cm^{-3}. Heildarþykkt kísilskífunnar er 300 μm.

(a) (4) Finna skal mesta As þéttleika, N_p.

(b) (4) Rissið upp þversnið As of B íbótar. Merkið greinilega inn á grafið skotlengd og hæsta þéttleika. Gera skal ráð Gaussísku þversniði.

(c) (3) Finna skal dýptina(irnar) niður að samskeytum rétt eftir ígræðsluna.

(d) (4) Nú er sýnið hitað upp í 950 °C í brot af sekúndu. Rissið nú upp þversnið arsens og bór eftir þessa háhita meðferð.

(e) (3) Sýnið er hitað í 1000 °C í langan tíma, segjum tvær vikur. Dragið nú aftur þversnið As og B.

(f) (3) Channeling getur einnig haft áhrif á þversnið íbótar eftir jónaígræðslunar. Útskýrið stuttlega channeling hrifin og listið/lýsið aðferðum til að draga úr áhrifum (að mestu fjórar setningar).

(g) (3) Það að mynda ofur-grunn skeyti sem lind/svelg framlengingar í nanóskala MOSFETum er enn virkt rannsóknarsvið. Ræðið hvort það að mynda ofur grunn skeyti er erfiðara fyrir p eða n íbót. Réttlætið svarið stuttlega (þrjár setningar að mestu).

An ion implantation step (dose = 10^{14} cm^{-2}, $R_\mathrm{p} = 0.0226$ μm, $\Delta R_\mathrm{p} = 0.0102$ μm) implants Arsenic (As) into a p-type semiconductor material with uniformly doped borron (B) background concentration of 10^{16} cm^{-3}. The total thickness of this Si wafer is 300 μm.

(a) (4) Find the peak As concentration, N_p.

(b) (4) Qualitatively draw the As and B concentration profiles. Clearly indicate and label the projected range and the peak conceentration. Assume an ideal Gaussian profile.

(c) (3) Find the junction depth(s) right after the implantation.

(d) (4) The sample is now thermally annealed at 950 °C for a fraction of a second. Qualitatively draw the Arsenic and boron profiles of this sample after high temperature annealing.

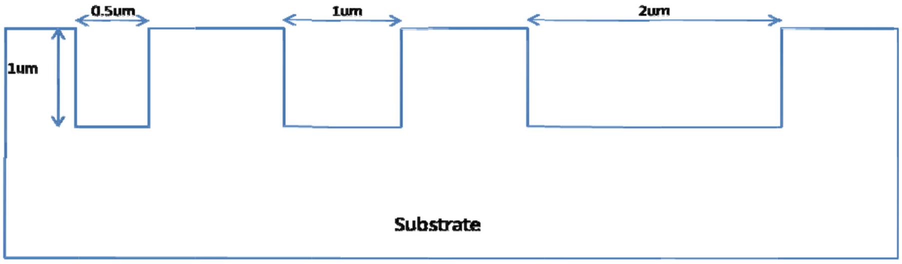
(e) (3) We do high temperature annealing at a 1000 °C for a long time, say two weeks. Redraw the As and B profiles.

(f) (3) Besides the various effects covered in the previous parts of this question, channeling could also affect the dopant profile after the ion implantation step. Briefly explain the channeling effect and list/describe two approaches for reducing the channeling effect (4 sentences max).

(g) (3) Achieving ultrashallow junctions as the source/drain extensions of nanoscale MOSFETs is a challenging field of active research. Speculate whether enabling ultrashallow junctions is more difficult for p$^+$ or n$^+$ doping. Briefly justify your answer (3 sentences max).

2. (15) **Ræktun húðar – Deposition of thin film**

(a) (3) Gerum ráð fyrir þversniði undirlags eins og sýnt er hér að neðan. Við ræktun á Al_2O_3 er góðri skrefþekju náð með ræktunarhraða 0.1 micron/min. Rissið þverskurð ræktaðrar húðar ef gert er ráð fyrir fullkominni skrefþekju eftir 1 min, 2 min, og 4 min ræktun.



(b) (6) Fyrir efnagufuágræðslu fjölkristallaðs kísils með $SiCl_4$ sem gaslind, er massaflutningsstuðullinn $h_g = 1$ cm/sec, hraðafasti yfirborðshvarfa $k_s = 2 \times 10^6 \exp(-1.9/kT)$ cm/sec, og þéttleiki Si atóma í gasflæðinu er $C_g = 3 \times 10^{16}$ atoms/cm^3. (Atómþéttleiki storkins kísilkristalls er 5×10^{22} atoms/cm^3.)

(i) (3) Hver er ræktunarhraðinn við 500 °C ?

(ii) (3) Við hvaða hitastig er $k_s = h_g$?

(c) (3) Listið upp meginkostina við það að nota efnagufuágræðslu (CVD) umfram physical vapor deposition (PVD) þegar ræktaðar eru þunnar húðir.

(d) (3) Listið upp megin kostina á því að nota spætun umfram uppgufun við ræktun á þunnum húðum.

(a) (3) Assume a starting substrate profile shown below. A conformal deposition of Al_2O_3 is then performed with a deposition rate of 0.1 micron/min. Sketch the cross sections of the deposited film for a completely conformal deposition after 1 min, 2 min, and 4 min of deposition.

(b) (6) For chemical vapor deposition of poly-Si using $SiCl_4$ as a gaseous source, the vaporphase mass transfer coefficient $h_g = 1$ cm/sec, the surface reaction rate constant $k_s = 2 \times 10^6 \exp(-1.9/kT)$ cm/sec, and the concentration of Si atoms in the gas stream $C_g = 3 \times 10^{16}$ atoms/cm^3. (The atomic concentration of solid Si is 5×10^{22} atoms/cm^3.)

(i) (3) What is the growth rate at 500 °C ?

(ii) (3) At what temperature does $k_{\mathrm{s}} = h_{\mathrm{g}}$?

(c) (3) List the major advantages of using chemical vapor deposition (CVD) versus physical vapor deposition (PVD) for thin films.

(d) (3) List the major advantages of using sputtering deposition versus evaporation deposition for thin films.

3. (10) **Oxun – Oxidation**

   Fyrir tiltekið oxunarferli, er það þekkt að oxunarhraðinn (dx_{ox}/dt) er 0.24 μm/klst þegar þykkt oxíðsins er 0.5 μm og verður 0.133 μm/klst þegar þykkt oxíðsins er 1 μm. Finna skal línulegan oxunar fasta (B/A) og fleygboga oxunarfasta B. Gefa skal svarið í viðeigandi einingum.

   For a particular oxidation process, it is known that the oxidation rate (dx_{ox}/dt) is 0.24 μm/hour when the oxide thickness is 0.5 μm and it becomes 0.133 μm/hour when the oxide thickness is 1 μm. Find the linear oxidation constant (B/A) and the parabolic oxidation constant B. Give answers in proper units.

4. (5) **Lithography**

   Fyrir tiltekið lithography ferli sem byggt er a vörpun, er minnsta upplausn $l_{\text{m}} = 1$ μm og depth of focus (DOF) er 1 μm. Ef að ljósopið yfir varpara linsunni er minnkað, ljósopið er helmingað, skal reikna ný gildi á l_{m} og DOF.

   For a particular lithography process based on projection printing, the minimum resolution (l_{m}) is 1 mum and the depth of focus (DOF) is 1 μm. By placing a smaller aperture over the projection lens, the numerical aperture (NA) is reduced by a factor of 2, calculate the new values of l_{m} and DOF.

5. (5) **Forsveim – Diffusion Predeposition**

   Forsveimi arsens er beitt til að mynda grunn skeyti í p-leiðandi kísli ($N_{\text{B}} = 1 \times 10^{15}$ cm^3). Leysnimörk As í kísli eru þekkt 1×10^{21} /cm^3. Ef að skeytin skulu vera ekki meira en 0.1 μm, hvert er lægsta sheet viðnám sem hægt er að fá fram ? Notið Irvin ferla.

   Diffusion predeposition of Arsenic is used to form a shallow junction in p-type Si ($N_{\text{B}} = 1 \times 10^{15}$ cm^3). The solid solubility of As in Si is known to be 1×10^{21} /cm^3. If the allowed junction depth is less than 0.1 μm, what is the lowest sheet resistance which can be achieved ? Use Irvin curves.

6. (5) **Lithography**

   Útskýrið hvers vegna kröfu um minnstu upplausn og mesta depth of focus er ekki hægt að besta samtímis með því að stytta bylgjulengd ljóseinda í vörpunar lithography.

   Explain why minimum resolution and depth of focus requirements cannot be optimized simultaneously by using shorter wavelength photons for projection optical lithography.

7. (6) **Kísill – Silicon**

   Af hverju er kísill mikilvægasta frumefnið á smárásum ? Nefnið þrjú atriði.

   Why is silicon the most important element in integrated circuits ? Name three reasons.

8. (8) **Valvísi í ætingu – Selectvity in etching**

   Finna skal hvaða valvísi skal krafist þægar ætt er 460 nm laga af fjölkristölluðum kísli án þess að æta meira en 3 nm niður í undirliggjandi gáttaroxíð. Gera skal ráð fyrir að einsleitni ætingar á fjölkristölluðum kísli sé 12 %.

   Find the etch selectivity required to etch a 460 nm polysilicon layer without removing more than 3 nm of its underlying gate oxide, assuming that the polysilicon is etched with a process having a 12 % etch rate uniformity.

9. (10) **Vandamál með ál leiðara – Problems with aluminum conductors**

   Á meðfylgjandi mynd sjást sum megin vandamálin sem fylgja því að nota ál sem leiðara í fjöllaga málmtengi. Lýstu ástæðum eftirfarandi vandamála og þeim lausnum sem beitt er til að draga úr eða eyða þessum vandamálum.

   (a) Hólar og innflákar (e. hillock and void)

   (b) Myndun álbrodda

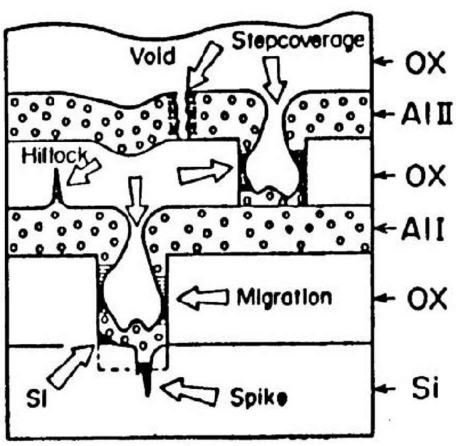
   (c) Ónóg skrefþekja (e. step coverage)

   The figure below shows some of the problems that arise when aluminum is used as a conductor in multilevel interconnects. Describe the reasons for the following problems and the soultions that are applied to minimize or eliminate the problems.

   (a) Hillocks and voids

   (b) Formation of aluminum spikes

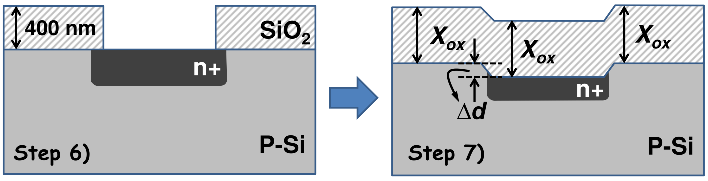
   (c) Limited step coverage



10. (12) **Oxun – Oxidation**

Gerum ráð fyrir að atómþéttleiki í hreinum kísli sé 5×10^{22} cm^{-3} og að sameindaþéttleiki SiO_2 sé 2.2×10^{22} cm^{-3}. Strúktúrinn hér að neðan hefur farið í gengum eftirfarandi frmaleiðsluskref:

Assume the atom density of pure Si is 5×10^{22} cm^{-3} and the molecular density of SiO_2 is 2.2×10^{22} cm^{-3}. The structure shown below has gone through the following process steps:

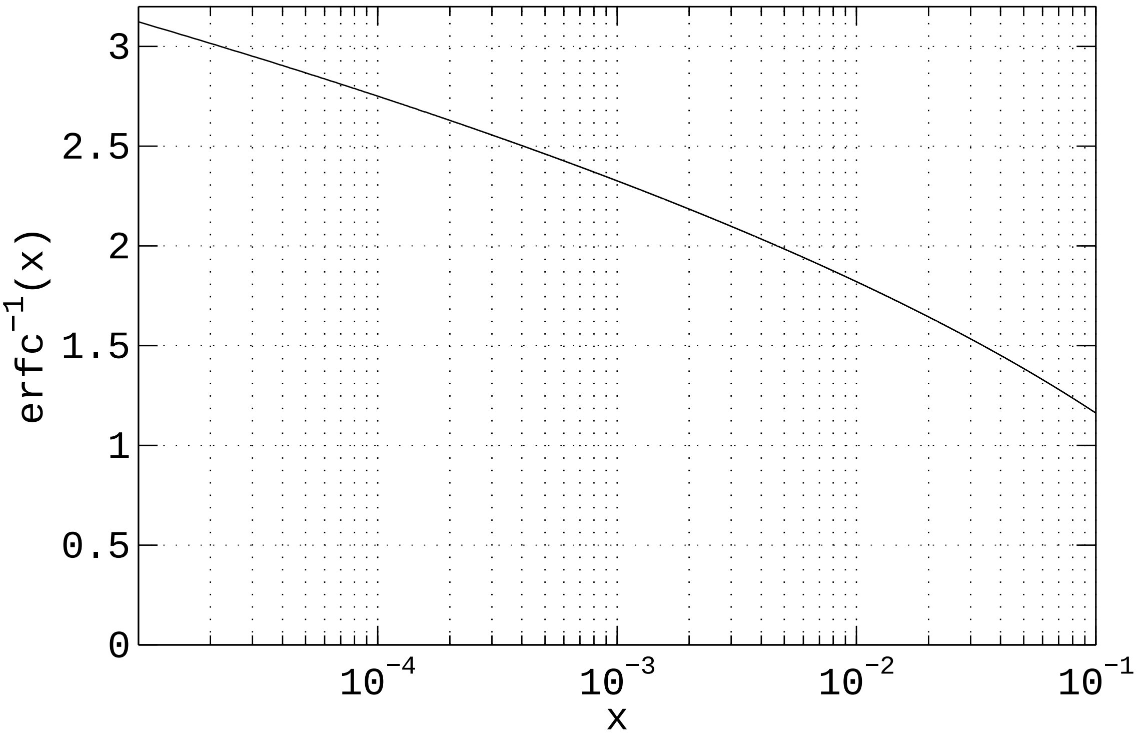


1) Start with a bare <100> silicon wafer
2) Grow 400 nm of oxide
3) Lithography
4) Dry etch 400 nm of oxide
5) Remove P.R.
6) Diffuse phosphorous (n+)
7) Wet oxidation for *t* minutes

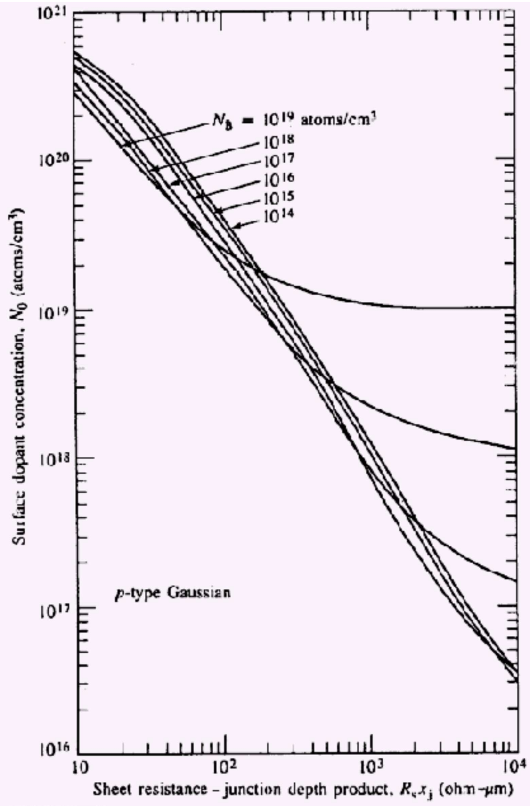
Gerum ráð fyrir að línulegur hraðafasti oxunar sé $(B/A)_{n^+} = 4 \times (B/A)_p = 0.4$ μm/klst (þ.e., yfirborðshraðafastinn eykst þegar fosfór er fyrir hendi), og fleygboga hraðafastar eru $(B)_{n^+} = (B)_p = 0.2$ μm^2/klst. Ákvarða t, Δd, og X_{ox}.

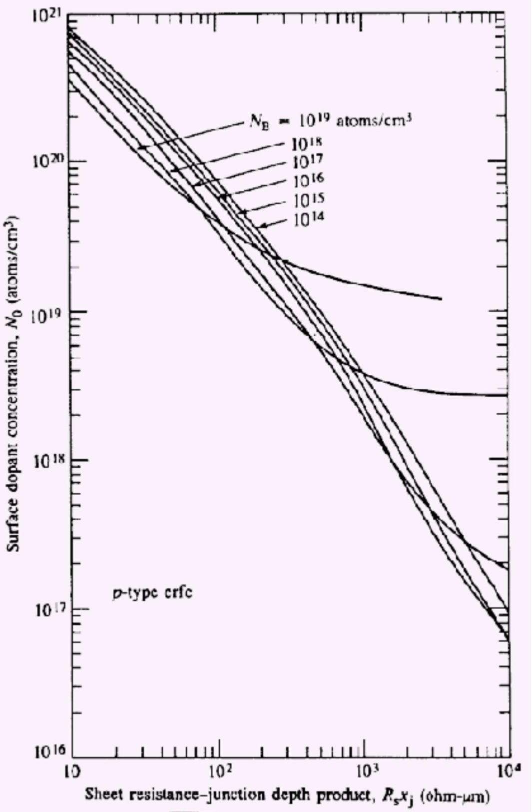
Assume that the linear oxidation rate constants are $(B/A)_{n^+} = 4 \times (B/A)_p = 0.4$ μm/hr (i.e., the surface reaction rate increases when phosphorous is present), and the parabolic constants are $(B)_{n^+} = (B)_p = 0.2$ μm^2/hr. Determine t, Δd, and X_{ox}.

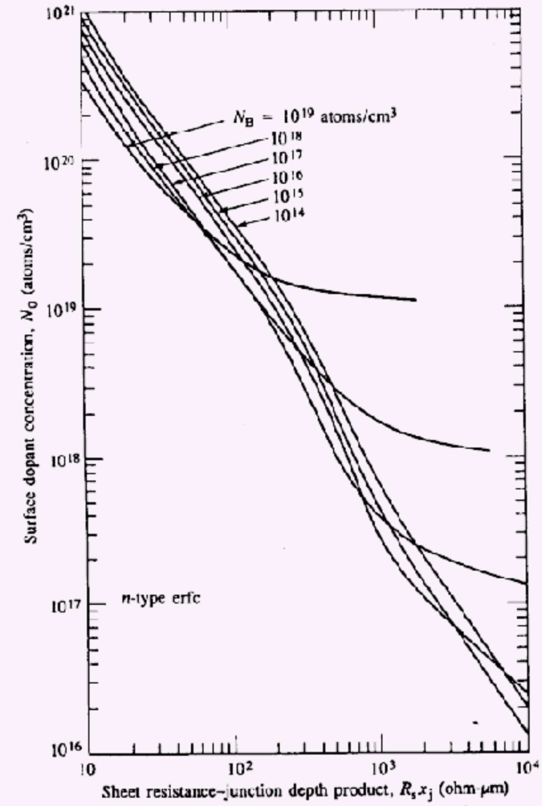
Gagnleg gröf

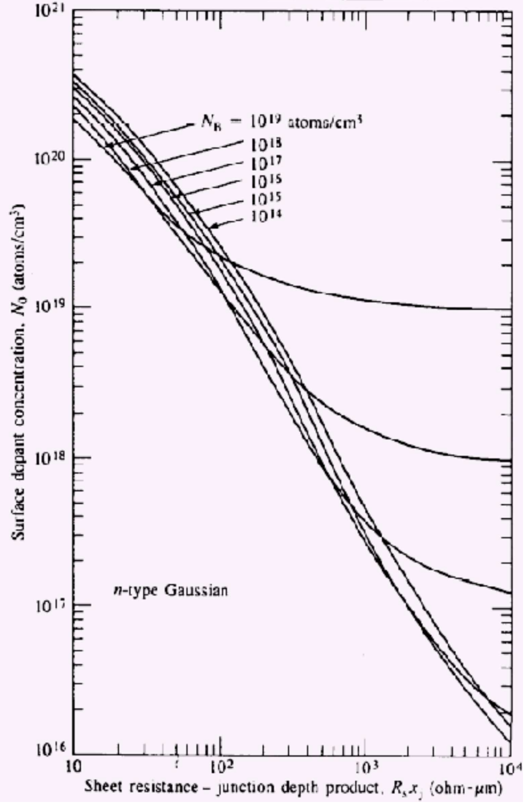


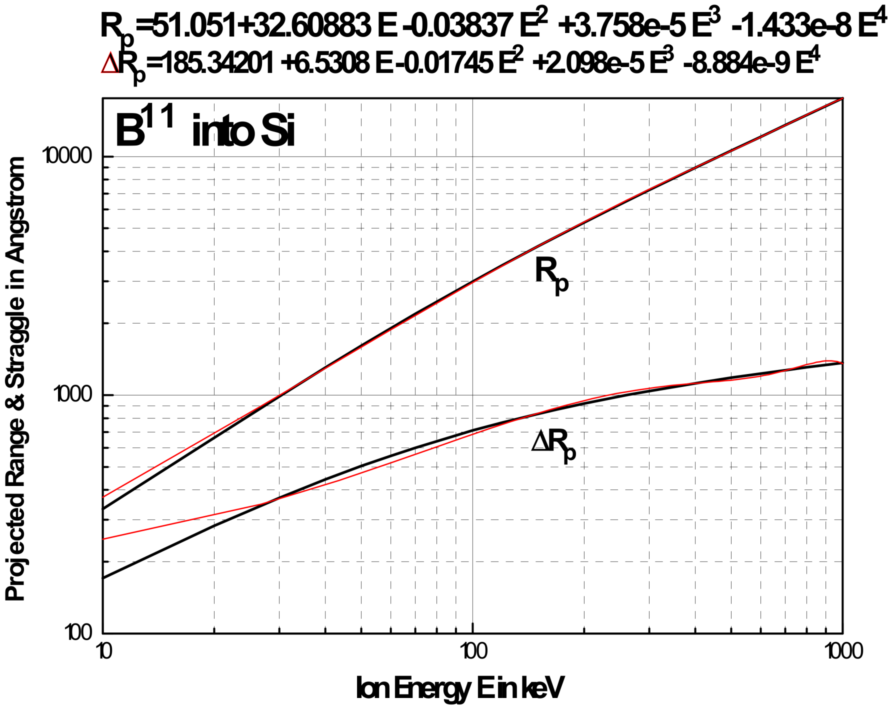
Irvin curves





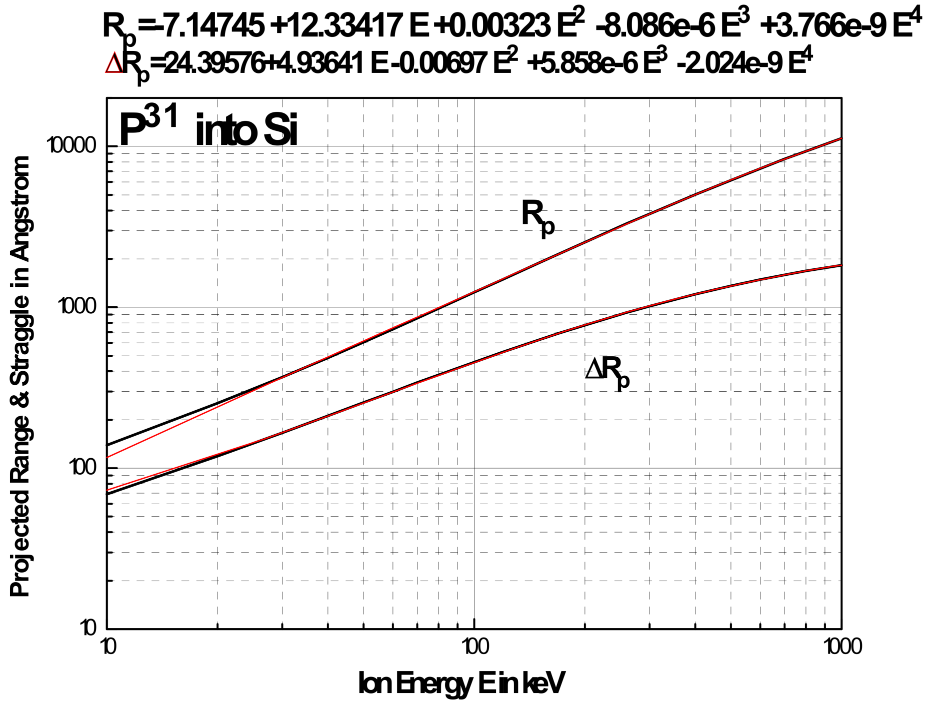






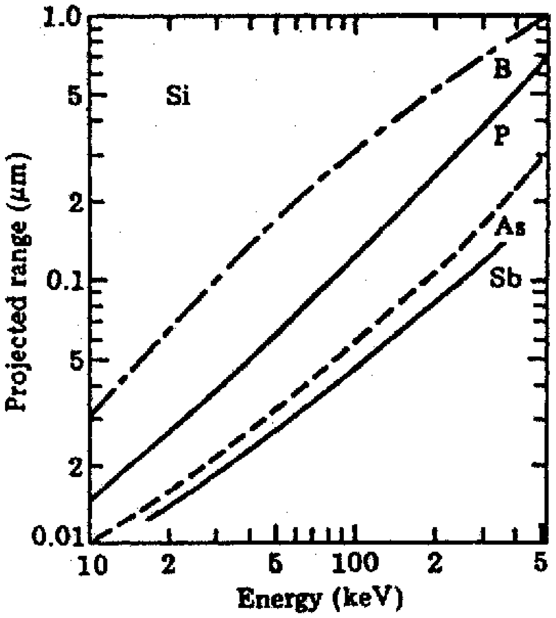
$R_p = 51.051 + 32.60883\,E - 0.03837\,E^2 + 3.758e\text{-}5\,E^3 - 1.433e\text{-}8\,E^4$

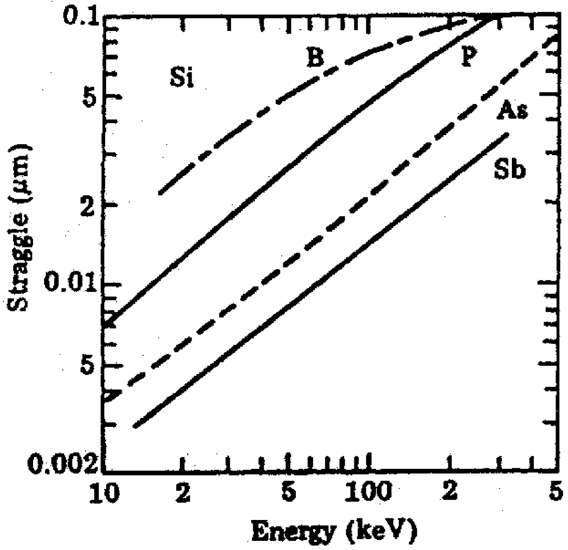
$\Delta R_p = 185.34201 + 6.5308\,E - 0.01745\,E^2 + 2.098e\text{-}5\,E^3 - 8.884e\text{-}9\,E^4$



$R_p = -7.14745 + 12.33417\,E + 0.00323\,E^2 - 8.086e\text{-}6\,E^3 + 3.766e\text{-}9\,E^4$

$\Delta R_p = 24.39576 + 4.93641\,E - 0.00697\,E^2 + 5.858e\text{-}6\,E^3 - 2.024e\text{-}9\,E^4$


1.0
5
2
0.1
5
2
0.01
Projected range (μm)
Si
B
P
As
Sb
10
2
5
100
2
5
Energy (keV)


0.1
5
2
0.01
5
0.002
Straggle (μm)
Si
B
P
As
Sb
10
2
5
100
2
5
Energy (keV)

1 Fastar

$$q = 1.602 \times 10^{-19} \quad \mathrm{C}$$

$$N_{\mathrm{Av}} = 6.022 \times 10^{23} \quad \text{sameindir/mól}$$

$$k = 1.38 \times 10^{-23} \mathrm{J/K} = 8.62 \times 10^{-5} \quad \mathrm{eV/K}$$

$$\epsilon_0 = 8.854 \times 10^{-14} \quad \mathrm{F/cm}$$

$$\epsilon_{\mathrm{ox}}/\epsilon_0 = 3.9$$

$$\epsilon_{\mathrm{Si}}/\epsilon_0 = 11.9$$

$$\epsilon_{\mathrm{Ge}}/\epsilon_0 = 16$$

$$\epsilon_{\mathrm{GaAs}}/\epsilon_0 = 13.1$$

Fyrir kísil við stofuhita:

$$n_{\mathrm{i}} = 9.65 \times 10^{9} \quad \mathrm{cm}^{-3}$$

Fyrir GaAs við stofuhita:

$$n_{\mathrm{i}} = 2.25 \times 10^{9} \quad \mathrm{cm}^{-3}$$

2 Ræktun

$$k_0 = \frac{C_{\mathrm{s}}}{C_{\mathrm{l}}}$$

$$C_{\mathrm{s}} = k_0 C_0 \left[1 - \frac{M}{M_0}\right]^{k_0 - 1}$$

3 Hálfleiðarar

$$E_{\mathrm{H}} = -\frac{m_{\mathrm{e}} q^4}{8\epsilon_0^2 h^2 n^2} = -\frac{13.6}{n^2}$$

$$E_{\mathrm{g}} = 1.17 - \frac{(4.73 \times 10^{-4}) T^2}{(T + 636)} \qquad \text{kísill}$$

$$E_{\mathrm{g}} = 1.52 - \frac{(5.4 \times 10^{-4}) T^2}{(T + 204)} \qquad \text{GaAs}$$

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_{\mathrm{F}}}{kT}\right)}$$

$$n = \int_{\infty}^{E_{\mathrm{c}}} f(E) N(E) dE$$

$$N(E) = 4\pi \left(\frac{2m^*}{h^2}\right)^{3/2} E^{1/2}$$

$$f(E) \approx \exp\left(-\frac{E - E_{\mathrm{F}}}{kT}\right) \text{ ef } E - E_{\mathrm{F}} > 3kT$$

$$f(E) \approx 1 - \exp\left(-\frac{E_{\mathrm{F}} - E}{kT}\right) \text{ ef } E - E_{\mathrm{F}} < 3kT$$

$$n \approx N_{\mathrm{c}} \exp\left(-\frac{E_{\mathrm{c}} - E_{\mathrm{F}}}{kT}\right)$$

$$N_{\mathrm{c}} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$p \approx N_{\mathrm{v}} \exp\left(-\frac{E_{\mathrm{F}} - E_{\mathrm{v}}}{kT}\right)$$

$$N_{\mathrm{v}} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$np = N_\mathrm{c} N_\mathrm{v} \exp\left(-\frac{E_\mathrm{g}}{kT}\right) = n_\mathrm{i}^2$$

$$n = n_\mathrm{i} \exp\left(\frac{E_\mathrm{F} - E_\mathrm{i}}{kT}\right)$$

$$p = n_\mathrm{i} \exp\left(\frac{E_\mathrm{i} - E_\mathrm{F}}{kT}\right)$$

$$E_\mathrm{c} - E_\mathrm{F} = kT \ln\left(\frac{N_\mathrm{c}}{N_\mathrm{D}}\right)$$

$$E_\mathrm{F} - E_\mathrm{v} = kT \ln\left(\frac{N_\mathrm{v}}{N_\mathrm{A}}\right)$$

$$np = n_\mathrm{i}^2$$

Við stofuhita fyrir kísil

$$N_\mathrm{c} = 2.8 \times 10^{19} \quad \mathrm{cm}^{-3}$$

$$N_\mathrm{v} = 1.04 \times 10^{19} \quad \mathrm{cm}^{-3}$$

Við stofuhita fyrir GaAs

$$N_\mathrm{c} = 4.7 \times 10^{17} \quad \mathrm{cm}^{-3}$$

$$N_\mathrm{v} = 7 \times 10^{18} \quad \mathrm{cm}^{-3}$$

n-leiðandi hálfleiðari

$$n_\mathrm{n} = \frac{1}{2}\left[N_\mathrm{D} - N_\mathrm{A} + \sqrt{(N_\mathrm{D} - N_\mathrm{A})^2 + 4n_\mathrm{i}^2}\right]$$

og

$$p_\mathrm{n} = \frac{n_\mathrm{i}^2}{n_\mathrm{n}}$$

p-leiðandi hálfleiðari

$$p_\mathrm{p} = \frac{1}{2}\left[N_\mathrm{A} - N_\mathrm{D} + \sqrt{(N_\mathrm{A} - N_\mathrm{D})^2 + 4n_\mathrm{i}^2}\right]$$

og

$$n_\mathrm{p} = \frac{n_\mathrm{i}^2}{p_\mathrm{p}}$$

$$N_\mathrm{C} = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$N_\mathrm{V} = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$J = \sigma\mathcal{E}$$

$$\sigma = \frac{nq^2\tau}{m_\mathrm{n}^*} \qquad [\Omega\mathrm{cm}]^{-1}$$

$$\sigma = qn\mu_\mathrm{n}$$

$$\mu_\mathrm{n} = \frac{q\tau}{m_\mathrm{n}^*}$$

$$J = q(n\mu_\mathrm{n} + p\mu_\mathrm{p})\mathcal{E} = \sigma\mathcal{E}$$

$$R = \frac{\rho L}{Wd} = \frac{L}{Wd}\frac{1}{\sigma}$$

4 MOSFET

MOS kjörtvistur

$$q\phi_\mathrm{ms} = q(\phi_\mathrm{m} - \phi_\mathrm{s})$$

$$q\phi_\mathrm{ms} = q\phi_\mathrm{m} - \left[q\chi + \frac{E_\mathrm{g}}{2} + q\phi_\mathrm{b}\right]$$

$$Q_\mathrm{sc} = -qN_\mathrm{A}x_\mathrm{dmax} \approx -\sqrt{2q\epsilon_\mathrm{s}N_\mathrm{A}(2\psi_\mathrm{b})}$$

$$\psi_\mathrm{s}(\text{umhverfing}) \approx 2\psi_\mathrm{b} = \frac{2kT}{q}\ln\left(\frac{N_\mathrm{A}}{n_\mathrm{i}}\right)$$

$$x_{\mathrm{dmax}} = \left(\frac{2\epsilon_{\mathrm{s}}\psi_{\mathrm{s}}(\mathrm{umhv.})}{qN_{\mathrm{A}}}\right)^{1/2} \approx \left(\frac{2\epsilon_{\mathrm{s}}(2\psi_{\mathrm{b}})}{qN_{\mathrm{A}}}\right)^{1/2}$$

$$C_{\mathrm{ox}} = \frac{\epsilon_{\mathrm{ox}}}{d}$$

$$C_{\mathrm{d}} = \frac{\epsilon_{\mathrm{s}}}{x_{\mathrm{d}}}$$

$$C = \frac{C_{\mathrm{ox}}C_{\mathrm{d}}}{C_{\mathrm{ox}} + C_{\mathrm{d}}}$$

$$I_{\mathrm{D}} \approx \frac{W}{L}\mu_{\mathrm{n}}C_{\mathrm{ox}}(V_{\mathrm{G}} - V_{\mathrm{T}})V_{\mathrm{D}}$$

$$\text{ef} \quad V_{\mathrm{D}} \ll (V_{\mathrm{G}} - V_{\mathrm{T}})$$

$$V_{\mathrm{T}} \approx \frac{\sqrt{2\epsilon_{\mathrm{s}}qN_{\mathrm{A}}(2\psi_{\mathrm{b}})}}{C_{\mathrm{ox}}} + 2\psi_{\mathrm{b}}$$

$$V_{\mathrm{Dsat}} \approx V_{\mathrm{G}} - 2\psi_{\mathrm{b}} + K^2\left[1 - \left(1 + \frac{2V_{\mathrm{G}}}{K^2}\right)^{1/2}\right]$$

þar sem

$$K \equiv \frac{\epsilon_{\mathrm{Si}}qN_{\mathrm{A}}}{C_{\mathrm{ox}}}$$

$$I_{\mathrm{Dsat}} \approx \frac{W\mu_{\mathrm{n}}C_{\mathrm{ox}}}{2L}(V_{\mathrm{G}} - V_{\mathrm{T}})^2$$

Frávik frá kjörhegðan

$$V_{\mathrm{FB}} = \phi_{\mathrm{ms}} - \frac{Q_{\mathrm{it}}}{C_{\mathrm{ox}}}$$

$$V_{\mathrm{T}} = \phi_{\mathrm{ms}} - \frac{Q_{\mathrm{it}}}{C_{\mathrm{ox}}} - \frac{Q_{\mathrm{sc}}}{C_{\mathrm{ox}}} + 2\psi_{\mathrm{b}}$$

$$\psi_{\mathrm{b}} = (E_{\mathrm{i}} - E_{\mathrm{F}})/q$$

$$\Delta V_{\mathrm{T}} = \frac{\sqrt{2q\epsilon_{\mathrm{s}}N_{\mathrm{A}}}}{C_{\mathrm{o}}}\left[(2\psi_{\mathrm{b}} + V_{\mathrm{BS}})^{1/2} - (2\psi_{\mathrm{b}})^{1/2}\right]$$

$$x_{\mathrm{S}} = \left(\frac{2\epsilon_{\mathrm{s}}}{qN_{\mathrm{A}}}(V_{\mathrm{bi}} + V_{\mathrm{BS}})\right)^{1/2}$$

$$x_{\mathrm{D}} = \left(\frac{2\epsilon_{\mathrm{s}}}{qN_{\mathrm{A}}}(V_{\mathrm{D}} + V_{\mathrm{bi}} + V_{\mathrm{BS}})\right)^{1/2}$$

5 Viðnám

$$R = \frac{\rho L}{A}$$

$$\sigma = \frac{1}{\rho} = (q\mu_{\mathrm{n}}n + q\mu_{\mathrm{p}}p)$$

$$R = \frac{1}{G} = \frac{L}{W}\frac{1}{g}$$

6 Hreyfifræði gass

$$pV = RT = N_{\mathrm{Av}}kT$$

$$f_{\mathrm{v}} = \frac{4}{\sqrt{\pi}}\left(\frac{m}{2kT}\right)^{3/2} v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$

$$\phi = \frac{p}{(2\pi mkT)^{1/2}} = 3.51 \times 10^{22}\left(\frac{p}{\sqrt{MT}}\right)$$

7 Lagvöxtur

$$C_{\mathrm{s}} = \frac{C_{\mathrm{g}}}{1 + (k_{\mathrm{s}}/h_{\mathrm{g}})}$$

$$v = \frac{k_{\mathrm{s}}h_{\mathrm{g}}}{k_{\mathrm{s}} + h_{\mathrm{g}}}\left(\frac{C_{\mathrm{t}}}{C_{\mathrm{a}}}\right)y$$

$$\delta(x) \approx \sqrt{\frac{\mu x}{\rho_{\mathrm{d}}v}}$$

$$\bar{\delta}(x) = \frac{1}{L}\int_0^L \delta(x)dx = \frac{2}{3}\sqrt{\frac{\mu L}{\rho_{\mathrm{d}}v}}$$

$$h_{\mathrm{g}} = \frac{D_{\mathrm{g}}}{\delta} = \frac{3}{2}D_{\mathrm{g}}\sqrt{\frac{v\rho_{\mathrm{d}}}{\mu L}}$$

8 Oxun

$$x^2 + \frac{2D}{k}x = \frac{2DC_0}{C_1}(t+\tau)$$

$$\tau \equiv \left(d_0^2 + \frac{2Dd_0}{k}\right)\frac{C_1}{2DC_0}$$

$$x = \frac{D}{k}\left[\left(1 + \frac{2C_0k^2(t+\tau)}{DC_1}\right)^{1/2} - 1\right]$$

$$x^2 + Ax = B(t+\tau)$$

$$A \equiv \frac{2D}{k}$$

$$B \equiv \frac{2DC_0}{C_1}$$

$$\frac{B}{A} \equiv \frac{kC_0}{C_1}$$

9 Sveim

$$F = -D\frac{\partial C}{\partial x}$$

$$\frac{\partial C}{\partial t} = -\frac{\partial F}{\partial x} = \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right)$$

$$D = D_0 \exp\left(-\frac{E_\mathrm{a}}{kT}\right)$$

Fastur yfirborðsþéttleiki

$$C(x,t) = C_\mathrm{s}\mathrm{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$$

$$Q(t) = \int_0^\infty C(x,t)dx$$

$$Q(t) = \frac{2}{\sqrt{\pi}}C_\mathrm{s}\sqrt{Dt} \approx 1.13C_\mathrm{s}\sqrt{Dt}$$

Föst heildaríbót

$$\int_0^\infty C(x,t)dx = S$$

$$C(\infty,t) = 0$$

$$C(x,t) = \frac{S}{\sqrt{\pi Dt}}\exp\left(-\frac{x^2}{4Dt}\right)$$

$$C_\mathrm{s}(t) = \frac{S}{\sqrt{\pi Dt}}$$

10 Jónaígræðsla

$$\left(\frac{dE}{dx}\right)_\mathrm{tot} = S_\mathrm{n}(E) + S_\mathrm{e}(E)$$

$$\int_0^R dx = R = -\int_E^0 \frac{dE}{(dE/dx)_\mathrm{tot}}$$

$$S_\mathrm{e} = k_\mathrm{e}\sqrt{E}$$

$$N(x) = N_\mathrm{p}\exp\left[-\frac{(x-R_\mathrm{p})^2}{2(\Delta R_\mathrm{p})^2}\right]$$

$$Q = \int_0^\infty N(x)dx$$

$$Q = \sqrt{2\pi}N_\mathrm{p}\Delta R_\mathrm{p}$$

$$N(x) = \frac{N_\mathrm{p}}{\left[1 + \frac{4Dt}{2(\Delta R_\mathrm{p})^2}\right]^{1/2}}\exp\left[-\frac{(x-R_\mathrm{p})^2}{2(\Delta R_\mathrm{p})^2 + 4Dt}\right]$$

11 Málmar

$$\text{MTF} \sim \frac{1}{J^2} \exp\left(\frac{E_\text{a}}{kT}\right)$$

12 Lithography

$$\text{CD} = W_\text{min} \approx \sqrt{\lambda g}$$

$$R = \frac{0.61\lambda}{n \sin\theta}$$

$$R = \frac{0.61\lambda}{\text{NA}} = k_1 \frac{\lambda}{\text{NA}}$$

$$\text{NA} \equiv n \sin\theta$$

$$\text{DOF} = \pm\frac{R/2}{\tan\theta} \approx \pm\frac{R/2}{\sin\theta} = \pm k_2 \frac{\lambda}{(\text{NA})^2}$$

$$\text{MTF} = \frac{I_\text{MAX} - I_\text{MIN}}{I_\text{MAX} + I_\text{MIN}}$$

$$\text{CMTF}_\text{viðnám} = \frac{E_\text{T} - E_1}{E_\text{T} + E_1} = \frac{10^{1/\gamma} - 1}{10^{1/\gamma} + 1}$$

$$\gamma = \frac{1}{\log\left[\frac{E_\text{T}}{E_1}\right]}$$

13 Framleiðni

Líkan Poisson

$$Y = \frac{1}{\text{e}^{AD}}$$

Líkan Murphy

$$Y = \left[\frac{1 - \text{e}^{-AD}}{AD}\right]^2$$

Líkan Seeds

$$Y = \frac{1}{\text{e}^{\sqrt{AD}}}$$