Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2014

Eðlisfræði þéttefnis I

Dæmablað 6

Skilafrestur 7. October 2014 kl. 15:00

1. **Dispersion relation** (15)

   Consider 1D chain with identical masses M. Assume that there are nearest neighbor(nn) springs with spring constant K_1 and next nearest neighbor(nnn) springs with spring constant $K_2 < K_1$.

   (a) Find the dispersion relation for this system.

   (b) Calculate the speed of sound and compare to a system without nnn interactions.

   (c) Now generalize to a new energy:

$$U^{\text{harmonic}} = \sum_n \sum_{m>0} K_m \left[u(na) - u([n+m]\,a)\right]^2$$

   (i) Show that the dispersion relation is:

$$\omega = 2\sqrt{\sum_{m>0} K_m \frac{\sin^2(\frac{1}{2}mka)}{M}}$$

   (ii) Show that the long wavelength limit of the dispersion relation for $K_m = K_1/m^p$ when $p = 3$ is

$$\omega \sim k\sqrt{|\ln k|}$$

2. **Phonon dispersion with alternating spring** (20)

   Consider a one-dimensional chain of identical atoms. The springs between them alternate in strength between values K_1 and K_2.

(a) Find the vibrational frequencies as a function of wave number q. Study the low q limit and find the sound velocity.

(b) Discuss the physical meaning of the two branches. Sketch the way the atoms move in both cases !

(c) Discuss the dispersion and the normal modes for $K_1 \gg K_2$.

(d) Discuss the limit $K_1 \approx K_2$ and compare with the homogeneous chain where all springs are identical.