Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2014

Eðlisfræði þéttefnis I

Dæmablað 9

Skilafrestur 28. October 2014 kl. 15:00

1. **Anomalous density of states** (15)

   A certain two-dimensional simple-square lattice of lattice constant a has the dispersion relation $(\omega_{\rm L}(q))^2 = c_{\rm L}^2 q^2$, for longitudinal vibrations and $(\omega_{\rm T}(q))^2 = c_{\rm T}^2 q^2$ for transverse vibrations for $qa \ll 1$. The density of mode frequencies $D(\omega)$ determines the temperature dependence of the specific heat.

   (a) What is $D(\omega)$ for an $N-$atom crystal at frequencies ω described by the dispersion relation above ? That is, what is the number of modes between ω and $\omega + d\omega$? Suppose that for some wave-vector q_0, the $\omega_{\rm L}$ has an absolute maximum. Near this maximum $\omega_{\rm L}(q) = \omega_0 - A(q - q_0)^2$. You can assume $\omega_{\rm T}(q)$ always lies well below ω_0, so that these modes dont contribute to (b).

   (b) Find the form of $D(\omega)$ when $\omega \leq \omega_0$, and when $\omega \geq \omega_0$.

2. **van Hove singularities** (15)

   (a) In a linear harmonic chain with only nearest-neighbor interactions, the normal-mode dispersion relation has the form $\omega(q) = \omega_0|\sin(qa/2)|$, where the constant ω_0 is the maximum frequency (assumed when q is on the zone boundary). Show that the density of normal modes in this case is given by

$$D(\omega) = \frac{2}{\pi a\sqrt{\omega_0^2 - \omega^2}}.$$

   The singularity at $\omega_0 - \omega$ is called a van Hove singularity.

(b) In three dimensions the van Hove singularities are infinities not in the normal mode density itself, but in its derivative. Show that the normal modes in the neighborhood of a maximum of $\omega(q)$, for example, lead to a term in the normal mode density that varies as $(\omega_0 - \omega)^{1/2}$.

3. **Sound waves** (10)

   Would you expect to find sound waves in small maolucules ? If not, how do you explain the propagation of sound waves in gaseous substances ?