Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2015

Eðlisfræði þéttefnis I

Lokapróf

10. Desember 2015 kl. 09:00 - 12:00

Leyfileg hjálpargögn eru skriffæri, vasareiknir og eintak af kennslubók(um) (ein eða fleiri):

- Harald Ibach and Hans Lüth, *Solid-State Physics: An Introduction to Principles of Materials Science*, 4th ed., Springer-Verlag, 2009
- Steven H. Simon, *The Oxford Solid State Basics*, Oxford University Press, 2013
- Charles Kittel, *Introduction to Solid State Physcis*, John Wiley & Sons
- Neil W. Ashcroft and N. David Mermin, *Solid State Physics*, Brooks Cole, 1976
- M. Ali Omar, Elementary Solid State Physics: Principles and Applications, Addison-Wesley

en engar glósur eða dæmi.

1. **X-ray diffraction** (13)

NaCl kristallast í hliðarsetna teningsgrind þar sem grunnur Na og Cl eru aðskilinn með vegalengd sem er helmingur hornalínu teningsins. Atóm tölur Na og Cl eru 11 og 17.

(a) Ákvarða hvaða Röntgenspeglanir koma fram (merkt vísum fyrir dæmigerða teningsgrind).

(b) Af þessum hópum hver hópurinn gefur sterkt merki og hver veikt ?

NaCl crystallizes in a face-centered cubic lattice with a basis of Na and Cl ions seperated by half the body diagonal of the cube. The atomic numbers of Na and Cl are 11 and 17, respectively.

(a) Determine which X-ray reflections will be observed (indexed for the conventional cubic unit cell).

(b) Of these which group will be strong and which group weak ?

2. **Diatomic chain** (16)

Skoðum sveifluhætti í línulegri keðju þar sem kraftstuðlar milli næstu granna atóma eru til skiptis C og $10C$. Gerum ráð fyrir að massar atóma séu þeir sömu, og að fjarlægð milli næstu granna sé $a/2$. Finna skal $\omega(\mathbf{K})$ við $\mathbf{K} = 0$ og $\mathbf{K} = \pi/a$. Rissið tvístrunarsambandið. Þetta dæmi líkir eftir tvíatóma sameind eins og H_2.

Consider the normal modes of a linear chain in which the force constants between the nearest-neighbor atoms are alternately C and $10C$. Let the masses be equal, and let the nearest-neighbor separation be $a/2$. Find $\omega(\mathbf{K})$ at $\mathbf{K} = 0$ and $\mathbf{K} = \pi/a$. Sketch the dispersion relation. This problem simulates a crystal of diatomic molecules like H_2.

3. **Conduction mechanism of a solid** (18)

Myndin hér að neðan sýnir eðlisviðnám þéttefnis sem fall af hitastigi ($ErRhB_4$ óhreint, en ekki viljandi skemmt).

(a) Útfrá grafinu segið hvort þéttefnið er málmur eða einangrari.

(b) Lýsið megin ferlunum sem stýra leininni á eftirfarandi hitastigsbilum:

(1) T mjög nærri 0 K

(2) T nærri 25 K

(3) T nærri 300 K

(c) Áætlið meðal frjálsa vegalengd og meðal frjálsan tíma við $T = 0$ K og $T = 300$ K. Er þetta góður málmur ?

Nota má eftrifarandi gildi: $n = 10^{23}$ cm^{-3} og $v_{\mathrm{F}} = 10^8$ cm/s.

The figure below shows a rough plot of the electrical resistance of a solid ($ErRhB_4$ impure, but not purposely damaged).

(a) From the graph, is this material a metal or an insulator ?

(b) Describe the principal physical processes that account for the following three temperature regions:

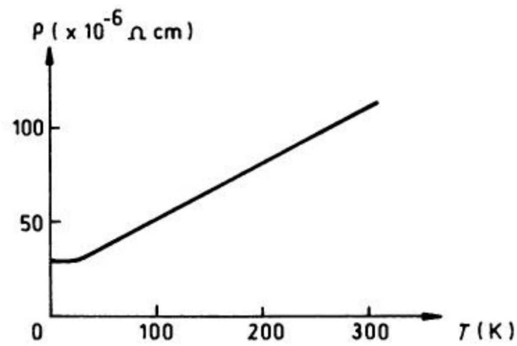
(1) T very near 0 K

(2) T near 25 K

(3) T near 300 K

(c) Estimate the mean free path and mean free time at $T = 0$ K and $T = 300$ K. Is the material a good metal ?

Useful parameters: $n = 10^{23}$ cm^{-3} og $v_{\mathrm{F}} = 10^8$ cm/s.



4. **Crystal potential in 1D lattice** (15)

Gerum ráð fyrir að kristallsmætti í einvíðri grind samanstandi af rétthyrndum brunnum umhverfis atómin. Gerum ráð fyrir að dýpt brunnanna sé V_0 og að breidd þeirra sé $a/5$.

(a) Beita skal nánast frjálsa rafeinda líkanið til að reikna stærð fyrstu þriggja orkugeilanna. Berið saman stærð þessara orkugeila.

(b) Reiknið orkugeilar fyrir tilfellin þegar $V_0 = 5$ eV og $a = 4$ Å.

Suppose that the crystal potential in a one-dimensional lattice is composed of a series of rectangular wells which surround the atom. Suppose the depth of each well is V_0 and its width $a/5$.

(a) Using the near free electron model, calculate the values of the first three energy gaps. Compare the magnitudes of these gaps.

(b) Evaluate these gaps for the case in which $V_0 = 5$ eV and $a = 4$ Å.

5. **Conductivity of a metal** (12)

Notið jöfnuna

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = -e\mathbf{E}$$

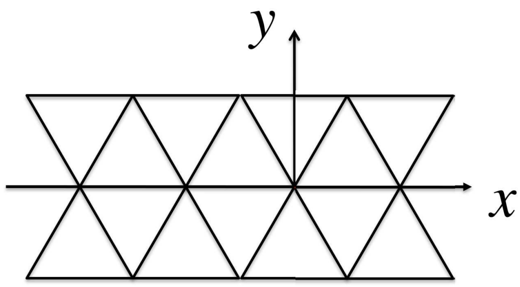
fyrir skriðþunga rafeinda til að finna AC leiðni málms. Svar þitt skal vera á forminu $\sigma(\omega, n, e, \tau)$.

Use the equation

$$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = -e\mathbf{E}$$

for the electron momentum to find the AC conductivity of a metal. Your answer should be of the form $\sigma(\omega, n, e, \tau)$.

6. **Two-dimensional triangular lattice – reciprocal lattice** (10)



(a) Merkið inn grunngrindareiningu í þessari tvívíðu þríhyrningsgrind. Finnið grunn vigranna.

(b) Finnið grunn vigra nykurgrindarinnar.

(a) Identify the primitive unit cell of a two-dimensional triangular lattice. Find the basis vectors.

(b) Construct the basis vectors of the reciprocal unit cell.

7. **Intrinsic semiconductor** (16)

Gerum ráð fyrir eiginleiðandi hálfleiðara. Látum $\mathcal{E}$ vera orku rafeindar. Látum $g_{\mathrm{C}}(\mathcal{E})$ vera ástandsþéttleika í leiðniborðanum, og $g_{\mathrm{V}}(\mathcal{E})$ vera ástandsþéttleika í gildisborðanum. Gerum ráð fyrir að $\mathcal{E}_{\mathrm{C}} - \mathcal{E}_{\mathrm{F}} \gg k_{\mathrm{B}}T$, $\mathcal{E}_{\mathrm{F}} - \mathcal{E}_{\mathrm{V}} \gg k_{\mathrm{B}}T$, og

$$g_{\mathrm{C}}(\mathcal{E}) = C_1(\mathcal{E} - \mathcal{E}_{\mathrm{C}})^{1/2}$$

$$g_{\mathrm{V}}(\mathcal{E}) = C_2(\mathcal{E}_{\mathrm{V}} - \mathcal{E})^{1/2}$$

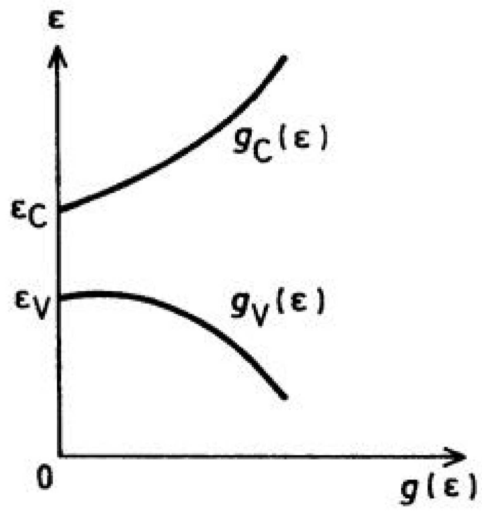
þar sem $\mathcal{E}_{\mathrm{C}}$ táknar orku við lágmark leiðniborða og $\mathcal{E}_{\mathrm{V}}$ orku við hámark gildisborða. Fermi orkan er $\mathcal{E}_{\mathrm{F}}$.

(a) Finna skal jöfnu fyrir n, rafeindaþéttleika í leiðniborðanum, sem fall af k_{B}, T, C_1, $\mathcal{E}_{\mathrm{C}}$, $\mathcal{E}_{\mathrm{F}}$ og einingarlausu ákveðnu tegri.

(b) Finna skal jöfnu fyrir p, holuþéttleika í gildisborðanum, sem fall af k_{B}, T, C_2, $\mathcal{E}_{\mathrm{V}}$, $\mathcal{E}_{\mathrm{F}}$ og einingarlausu ákveðnu tegri.

(c) Finna nákvæma jöfnu fyrir $\mathcal{E}_{\mathrm{F}}(T)$.

(d) Hver eða engin, af niðurstöðunum í (a), (b) eða (c) gildir ef hálfleiðarinn er íbættur með rafgjafa atómum ? Útskýrið.



Consider an intrinsic semiconductor. Let $\mathcal{E}$ be the energy of an electron. Let $g_{\mathrm{C}}(\mathcal{E})$ be the densty of states in the conduction band, and $g_{\mathrm{V}}(\mathcal{E})$ be the density of states in the valence band. Assume $\mathcal{E}_{\mathrm{C}} - \mathcal{E}_{\mathrm{F}} \gg k_{\mathrm{B}}T$, $\mathcal{E}_{\mathrm{F}} - \mathcal{E}_{\mathrm{V}} \gg k_{\mathrm{B}}T$, and

$$g_{\mathrm{C}}(\mathcal{E}) = C_1(\mathcal{E} - \mathcal{E}_{\mathrm{C}})^{1/2}$$

$$g_{\mathrm{V}}(\mathcal{E}) = C_2(\mathcal{E}_{\mathrm{V}} - \mathcal{E})^{1/2}$$

where $\mathcal{E}_{\mathrm{C}}$ represents the energy of the bottom of the conduction band and $\mathcal{E}_{\mathrm{V}}$ the top of the valence band. The Fermi energy is $\mathcal{E}_{\mathrm{F}}$.

(a) Find an expression for n, the number of electrons in the conduction band, in terms of k_{B}, T, C_1, $\mathcal{E}_{\mathrm{C}}$, $\mathcal{E}_{\mathrm{F}}$ and a dimensionless definite integral.

(b) Find an expression for p, the number of holes in the valence band, in terms of k_{B}, T, C_2, $\mathcal{E}_{\mathrm{V}}$, $\mathcal{E}_{\mathrm{F}}$ and a dimensionless definite integral.

(c) Find an explicit expression for $\mathcal{E}_{\mathrm{F}}(T)$.

(d) Which, if any, of the results of (a), (b) or (c) remain true if the material is doped with donor atoms ? Explain.

1 Fastar

$$q = 1.602 \times 10^{-19} \quad \text{C}$$

$$\hbar = 1.0546 \times 10^{-34} \quad \text{Js}$$

$$m_\text{e} = 9.1096 \times 10^{-31} \quad \text{Js}$$

$$N_\text{Av} = 6.022 \times 10^{23} \quad \text{sameindir/mól}$$

$$k_\text{B} = 1.38 \times 10^{-23}\text{J/K} = 8.62 \times 10^{-5} \quad \text{eV/K}$$

$$\epsilon_0 = 8.854 \times 10^{-14} \quad \text{F/cm}$$

$$\epsilon_\text{ox}/\epsilon_0 = 3.9$$

$$\epsilon_\text{Si}/\epsilon_0 = 11.9$$

$$\epsilon_\text{Ge}/\epsilon_0 = 16$$

$$\epsilon_\text{GaAs}/\epsilon_0 = 13.1$$

Fyrir kísil við stofuhita:

$$n_\text{i} = 9.65 \times 10^9 \quad \text{cm}^{-3}$$

Fyrir GaAs við stofuhita:

$$n_\text{i} = 2.25 \times 10^9 \quad \text{cm}^{-3}$$

2 Hálfleiðarar

$$E_\text{H} = -\frac{m_\text{e} q^4}{8\epsilon_0^2 h^2 n^2} = -\frac{13.6}{n^2}$$

$$E_\text{g} = 1.17 - \frac{(4.73 \times 10^{-4})T^2}{(T + 636)} \qquad \text{kísill}$$

$$E_\text{g} = 1.52 - \frac{(5.4 \times 10^{-4})T^2}{(T + 204)} \qquad \text{GaAs}$$

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_\text{F}}{kT}\right)}$$

$$n = \int_\infty^{E_\text{c}} f(E)N(E)dE$$

$$N(E) = 4\pi \left(\frac{2m^*}{h^2}\right)^{3/2} E^{1/2}$$

$$f(E) \approx \exp\left(-\frac{E - E_\text{F}}{kT}\right) \text{ ef } E - E_\text{F} > 3kT$$

$$f(E) \approx 1 - \exp\left(-\frac{E_\text{F} - E}{kT}\right) \text{ ef } E - E_\text{F} < 3kT$$

$$n \approx N_\text{c} \exp\left(-\frac{E_\text{c} - E_\text{F}}{kT}\right)$$

$$N_\text{c} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$p \approx N_\text{v} \exp\left(-\frac{E_\text{F} - E_\text{v}}{kT}\right)$$

$$N_\text{v} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$np = N_\text{c} N_\text{v} \exp\left(-\frac{E_\text{g}}{kT}\right) = n_\text{i}^2$$

$$n = n_\text{i} \exp\left(\frac{E_\text{F} - E_\text{i}}{kT}\right)$$

$$p = n_\text{i} \exp\left(\frac{E_\text{i} - E_\text{F}}{kT}\right)$$

$$E_\text{c} - E_\text{F} = kT \ln\left(\frac{N_\text{c}}{N_\text{D}}\right)$$

$$E_\text{F} - E_\text{v} = kT \ln\left(\frac{N_\text{v}}{N_\text{A}}\right)$$

$$np = n_\mathrm{i}^2$$

Við stofuhita fyrir kísil

$$N_\mathrm{c} = 2.8 \times 10^{19} \quad \mathrm{cm}^{-3}$$

$$N_\mathrm{v} = 1.04 \times 10^{19} \quad \mathrm{cm}^{-3}$$

Við stofuhita fyrir GaAs

$$N_\mathrm{c} = 4.7 \times 10^{17} \quad \mathrm{cm}^{-3}$$

$$N_\mathrm{v} = 7 \times 10^{18} \quad \mathrm{cm}^{-3}$$

n-leiðandi hálfleiðari

$$n_\mathrm{n} = \frac{1}{2}\left[N_\mathrm{D} - N_\mathrm{A} + \sqrt{(N_\mathrm{D} - N_\mathrm{A})^2 + 4n_\mathrm{i}^2}\right]$$

og

$$p_\mathrm{n} = \frac{n_\mathrm{i}^2}{n_\mathrm{n}}$$

p-leiðandi hálfleiðari

$$p_\mathrm{p} = \frac{1}{2}\left[N_\mathrm{A} - N_\mathrm{D} + \sqrt{(N_\mathrm{A} - N_\mathrm{D})^2 + 4n_\mathrm{i}^2}\right]$$

og

$$n_\mathrm{p} = \frac{n_\mathrm{i}^2}{p_\mathrm{p}}$$

$$N_\mathrm{C} = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$N_\mathrm{V} = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$J = \sigma\mathcal{E}$$

$$\sigma = \frac{nq^2\tau}{m_\mathrm{n}^*} \qquad [\Omega\mathrm{cm}]^{-1}$$

$$\sigma = qn\mu_\mathrm{n}$$

$$\mu_\mathrm{n} = \frac{q\tau}{m_\mathrm{n}^*}$$

$$J = q(n\mu_\mathrm{n} + p\mu_\mathrm{p})\mathcal{E} = \sigma\mathcal{E}$$

$$R = \frac{\rho L}{Wd} = \frac{L}{Wd}\frac{1}{\sigma}$$

3 Viðnám

$$R = \frac{\rho L}{A}$$

$$\sigma = \frac{1}{\rho} = (q\mu_\mathrm{n} n + q\mu_\mathrm{p} p)$$

$$R = \frac{1}{G} = \frac{L}{W}\frac{1}{g}$$