Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2016

Eðlisfræði þéttefnis I

Lokapróf

5. Desember 2016 kl. 13:30 – 16:30

Leyfileg hjálpargögn eru skriffæri, vasareiknir og eintak af kennslubók(um) (ein eða fleiri):

- Harald Ibach and Hans Lüth, *Solid-State Physics: An Introduction to Principles of Materials Science*, 4th ed., Springer-Verlag, 2009
- Steven H. Simon, *The Oxford Solid State Basics*, Oxford University Press, 2013
- Charles Kittel, *Introduction to Solid State Physcis*, John Wiley & Sons
- Neil W. Ashcroft and N. David Mermin, *Solid State Physics*, Brooks Cole, 1976
- M. Ali Omar, Elementary Solid State Physics: Principles and Applications, Addison-Wesley

en engar glósur eða dæmi.

1. **Röntgen bylgjubeygja – X-ray diffraction** (12)

   Pólon er eina frumefnið sem kristallast í einfaldan tening. Í bognunartilraun með Röntgengeislum af bylgjulengd 0.1789 nm, beygir fyrstu gráðu toppur, sem tengdur er (110) kristallaplaninu, um hornið 44.51°.

   (a) Hver er grindarfasti pólons ?

   (b) Undir hvaða horni væntir þú þess að finna fyrstu gráðu topp sem svarar til (111) plansins ?

   (c) Undir hvaða horni væntir þú þess að finna annarar gráðu topp sem svarar til (111) plansins ?

   (d) Í bognunartilraun með rafeindum er 200 keV rafeindageisli notaður til að skoða þunna pólon húð. Reikna hornið sem toppur frá (110) planinu kemur undir.

   Polonium is the only element that crystallises in a simple cubic structure. In a diffraction experiment using X-rays of wavelength 0.1789 nm, a first order diffraction peak associated with the (110) crystal plane is deflected through an angle of 44.51°.

   (a) What is the lattice constant of polonium ?

   (b) At what angle would you expect to find the first order peak corresponding to the (111) plane ?

   (c) At what angle would you expect to find the second order peak corresponding to the (111) plane ?

   (d) In an electron diffraction experiment, an electron beam with an energy of 200 keV is used to probe a thin foil of polonium. Calculate the angle through which the (110) diffraction peak is deflected.

2. **Röntgen bylgjubognun – X-ray diffraction** (9)

Grindarfasti (lengd tenings) einsatóma bcc kristalls er $a = 4.28$ Å. Reikna skal bylgjubognunarhorn 2θ fyrstu fjögurra toppa (þeirra fjögurra bylgjubognunartoppa sem hafa lægstu 2θ gildi) fyrir duft sýni, þegar beitt er einlitri Röntgen geislun með bylgjulengd $\lambda = 1.5$ Å. (duft sýni þýðir að allar kristallastefnur eru mögulegar í sýninu.)

The lattice constant (length of the conventional cubic cell) of a monatomic bcc crystal is $a = 4.28$ Å. Calculate the diffraction angles 2θ of the first four diffraction peaks (the four diffraction peaks with the lowest 2θ values) from its powder specimen, using monochromatic X-ray with a wavelength $\lambda = 1.5$ Å. (Hint: powder specimen implies that all crystal orientations are possible in the specimen.)

3. **Líkan Einstein fyrir eðlisvarma – Einstein's model for specific heat** (12)

Líkan Einstein fyrir þéttefni gefur jöfnu fyrir eðlisvarma

$$C_{\mathrm{v}} = 3N_0 k \left(\frac{\theta_{\mathrm{E}}}{T}\right)^2 \frac{\exp(\theta_{\mathrm{E}}/T)}{(\exp(\theta_{\mathrm{E}}/T) - 1)^2}$$

þar sem $\theta_{\mathrm{E}} = h\upsilon_{\mathrm{E}}/k$. Stuðullinn θ_{E} er nefndur hið einkennandi hitastig. Sýna skal

(a) að fyrir há hitastig fæst lögmál Dulong-Petit.

(b) að fyrir mjög lág hitastig fæst ekki T^3 lögmálið.

Einstein's model of solids gives the expression for the specific heat

$$C_{\mathrm{v}} = 3N_0 k \left(\frac{\theta_{\mathrm{E}}}{T}\right)^2 \frac{\exp(\theta_{\mathrm{E}}/T)}{(\exp(\theta_{\mathrm{E}}/T) - 1)^2}$$

where $\theta_{\mathrm{E}} = h\upsilon_{\mathrm{E}}/k$. The factor θ_{E} is called the characteristic temperature. Show that

(a) at high temperatures Dulong-Petit law is reproduced.

(b) at very low temperatures the T^3 law is not given.

4. **Periodic potential** (12)

Gerum ráð fyrir einvíðu rafeindakerfi sem hlítir veiku lotubundnu mætti

$$U(x) = U_0 \left[\cos^4 \left(\frac{\pi x}{a} \right) - \frac{3}{8} \right]$$

Ákvraðið margfeldni punktana við $k = \pm\pi/a$ og $k = \pm 2\pi/a$. Finnið og teiknið tvístrunarsambandið í fyrsta Brillouin svæðinu. Teiknið orkuna í einingunni $\hbar^2\pi^2/2ma^2$, bylgjuvigur í einingunni $1/a$, og gerið ráð fyrir að $U_0 = 0.1$ í þessum einingum.

Consider a one dimensional electron system subject to a weak periodic potential

$$U(x) = U_0 \left[\cos^4 \left(\frac{\pi x}{a} \right) - \frac{3}{8} \right]$$

Determine the degeneracy points at $k = \pm\pi/a$ and $k = \pm 2\pi/a$. Find and plot the dispersions of energy bands in the first Brillouin zone. Plot the energies in units of $\hbar^2\pi^2/2ma^2$, the wave numbers in units of $1/a$, and assume that $U_0 = 0.1$ in these units.

5. **Grafín bognunarháttur – Graphene bending mode** (14)

Til viðbótar við hina venjulegu hljóð- og ljóshætti, þá er í frístandandi grafín þynnu líka bognunarháttur. Þetta er þversum háttur með tvístrun $\omega(q) = aq^2$, þar sem a = fasti. Gera skal ráð fyrir að tvístrunarlögmálið gildi fyrir $0 \leq q \leq q_\mathrm{D}$. Finna skal framlag þessa bognunarháttar til eðlisvarmans í tveimur jaðar tilfellum:

(a) $k_\mathrm{B}T \gg \hbar\omega_\mathrm{D}$

(b) $k_\mathrm{B}T \ll \hbar\omega_\mathrm{D}$

þar sem $\omega_\mathrm{D} \equiv \omega(q_\mathrm{D})$.

In addition to the usual acoustic and optical modes, a free-standing membrane of graphene supports a bending mode. This is a transverse mode with dispersion $\omega(q) = aq^2$, where a = const. Assume that this dispersion law holds for $0 \leq q \leq q_\mathrm{D}$. Find the contribution of the bending mode to the specific heat in two limiting cases:

(a) $k_\mathrm{B}T \gg \hbar\omega_\mathrm{D}$

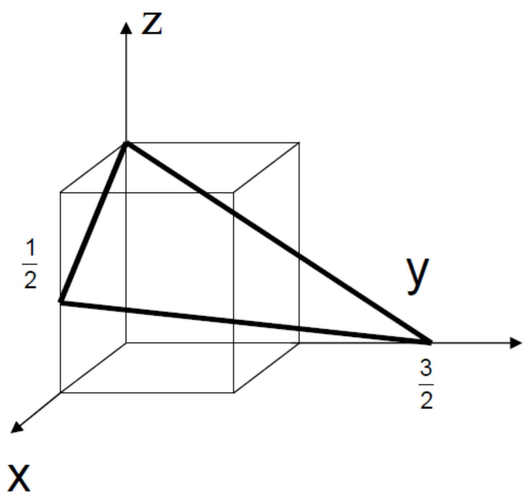
(b) $k_\mathrm{B}T \ll \hbar\omega_\mathrm{D}$

where $\omega_\mathrm{D} \equiv \omega(q_\mathrm{D})$.

6. **Miller vísir – Miller index** (5)

Hver er Miller vísir plansins á myndinni ?

What is the Miller index for the plane in the figure ?



7. **Hliðarsetinn teningur og bylgjubognun – Face centered cubic and diffraction** (18)

(a) Með hjálp teikningar, sýnið hvernig atómum er pakkað í hliðarsetna teningsgrind. Sýnið þétt pökkuðu plönin og Miller vísa þeirra.

(b) Grindarfasti einingargrindar miðjusetins kopar er 0.36 nm. Reikna skal lengstu bylgjulengd Röntgengeisla sem framkallar bylgjubognun frá þétt pökkuðu plönunum.

(c) Innkomandi Röntgengeisli er gefinn með $3.49 \times 10^{10}(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ m^{-1} og er beint á koparsýnið. Rita tilsvarandi bylgjuvgur fyrir útgangandi geisla, sem kemur frá þétt pakkaða planinu í lið (a).

(d) Hver er radíi koparatóms ? Gerið grein fyrir öllum nálgunum sem notaðar eru.

(e) Útskýrið hvers vegna ekki kemur fram bylgjubognun frá Röntgengeisla sem hefur bylgjulengd 0.6 nm.

(f) Myndurðu vænta þess að sjá bylgjubognun frá NaCl (grindarfasti einingargrindar er 0.56 nm) með Röntgengeisla af bylgjulengd 0.8 nm ? Réttlættu svar þitt.

(a) With the aid of a diagram, show how atoms are packed on a face-centered cubic lattice. Identify the close-packed planes and their Miller indices.

(b) The unit cell dimensions of face-centered cubic copper is 0.36 nm. Calculate the longest wavelength of X-rays that will produce diffraction from the closed-packed planes.

(c) An incoming X-ray beam given by $3.49 \times 10^{10}(2\mathbf{i} - \mathbf{j} + 2\mathbf{k})$ m^{-1} is incident on a copper sample. Write down the corresponding wavevector for an outgoing beam, diffracted by the close-packed plabe from part (a).

(d) Stating any assumptions that you use, what is the radius of a copper atom ?

(e) Explain why there is no diffraction from X-rays of 0.6 nm.

(f) Would you expect to see diffraction from NaCl (unit cell lattice constant 0.56 nm) using X-rays of wavelength 0.8 nm ? Justify your answer.

8. **Eiginleiðandi kísill – Intrinsic silicon** (9)

Stærð orkugeilar kísils er háð hitastigi samkvæmt

$$E_{\mathrm{g}} = 1.17\mathrm{eV} - 4.73 \times 10^{-4}\frac{T^2}{T + 636}.$$

Finna skal þéttleika rafeinda í leiðniborða eiginleiðandi (óíbættum) kísli við $T = 77$ K ef við 300 K $n_{\mathrm{i}} = 1.05 \times 10^{10}$ cm^{-3}.

The band gap of Si depends on the temperature as

$$E_{\mathrm{g}} = 1.17\mathrm{eV} - 4.73 \times 10^{-4}\frac{T^2}{T + 636}.$$

Find a concentration of electrons in the conduction band of intrinsic (undoped) Si at $T = 77$ K if at 300 K $n_{\mathrm{i}} = 1.05 \times 10^{10}$ cm^{-3}.

9. **Snertimætti p-n-skeyta – Built-in potential for a p-n-junction** (9)

Finna skal snertimætti fyrir kísil p-n skeyti við stofuhita ef boleðlisviðnám kísils er 1 Ω cm. Hreyfanleiki rafeinda í kísili er 1400 cm^2/Vs; $\mu_{\mathrm{n}}/\mu_{\mathrm{p}} = 3.1$; og $n_{\mathrm{i}} = 1.05 \times 10^{10}$ cm^{-3}.

Find the built-in potential for a p-n Si junction at room temperature if the bulk resistivity of Si is 1 Ω cm. Electron mobility in Si at RT is 1400 cm^2/Vs; $\mu_{\mathrm{n}}/\mu_{\mathrm{p}} = 3.1$; and $n_{\mathrm{i}} = 1.05 \times 10^{10}$ cm^{-3}.

1 Fastar

$$q = 1.602 \times 10^{-19} \quad \text{C}$$

$$\hbar = 1.0546 \times 10^{-34} \quad \text{Js}$$

$$m_\text{e} = 9.1096 \times 10^{-31} \quad \text{Js}$$

$$N_\text{Av} = 6.022 \times 10^{23} \quad \text{sameindir/mól}$$

$$k_\text{B} = 1.38 \times 10^{-23}\text{J/K} = 8.62 \times 10^{-5} \quad \text{eV/K}$$

$$\epsilon_0 = 8.854 \times 10^{-14} \quad \text{F/cm}$$

$$\epsilon_\text{ox}/\epsilon_0 = 3.9$$

$$\epsilon_\text{Si}/\epsilon_0 = 11.9$$

$$\epsilon_\text{Ge}/\epsilon_0 = 16$$

$$\epsilon_\text{GaAs}/\epsilon_0 = 13.1$$

Fyrir kísil við stofuhita:

$$n_\text{i} = 9.65 \times 10^9 \quad \text{cm}^{-3}$$

Fyrir GaAs við stofuhita:

$$n_\text{i} = 2.25 \times 10^9 \quad \text{cm}^{-3}$$

2 Hálfleiðarar

$$E_\text{H} = -\frac{m_\text{e} q^4}{8\epsilon_0^2 h^2 n^2} = -\frac{13.6}{n^2}$$

$$E_\text{g} = 1.17 - \frac{(4.73 \times 10^{-4})T^2}{(T + 636)} \qquad \text{kísill}$$

$$E_\text{g} = 1.52 - \frac{(5.4 \times 10^{-4})T^2}{(T + 204)} \qquad \text{GaAs}$$

$$m^* = \frac{\hbar^2}{d^2E/dk^2}$$

$$f(E) = \frac{1}{1 + \exp\left(\frac{E - E_\text{F}}{kT}\right)}$$

$$n = \int_\infty^{E_\text{c}} f(E)N(E)dE$$

$$N(E) = 4\pi \left(\frac{2m^*}{h^2}\right)^{3/2} E^{1/2}$$

$$f(E) \approx \exp\left(-\frac{E - E_\text{F}}{kT}\right) \text{ ef } E - E_\text{F} > 3kT$$

$$f(E) \approx 1 - \exp\left(-\frac{E_\text{F} - E}{kT}\right) \text{ ef } E - E_\text{F} < 3kT$$

$$n \approx N_\text{c} \exp\left(-\frac{E_\text{c} - E_\text{F}}{kT}\right)$$

$$N_\text{c} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$p \approx N_\text{v} \exp\left(-\frac{E_\text{F} - E_\text{v}}{kT}\right)$$

$$N_\text{v} = 2\left(\frac{2\pi m^* kT}{h^2}\right)^{3/2}$$

$$np = N_\text{c} N_\text{v} \exp\left(-\frac{E_\text{g}}{kT}\right) = n_\text{i}^2$$

$$n = n_\text{i} \exp\left(\frac{E_\text{F} - E_\text{i}}{kT}\right)$$

$$p = n_\text{i} \exp\left(\frac{E_\text{i} - E_\text{F}}{kT}\right)$$

$$E_\text{c} - E_\text{F} = kT \ln\left(\frac{N_\text{c}}{N_\text{D}}\right)$$

$$E_\text{F} - E_\text{v} = kT \ln\left(\frac{N_\text{v}}{N_\text{A}}\right)$$

$$np = n_\mathrm{i}^2$$

Við stofuhita fyrir kísil

$$N_\mathrm{c} = 2.8 \times 10^{19} \quad \mathrm{cm}^{-3}$$

$$N_\mathrm{v} = 1.04 \times 10^{19} \quad \mathrm{cm}^{-3}$$

Við stofuhita fyrir GaAs

$$N_\mathrm{c} = 4.7 \times 10^{17} \quad \mathrm{cm}^{-3}$$

$$N_\mathrm{v} = 7 \times 10^{18} \quad \mathrm{cm}^{-3}$$

n-leiðandi hálfleiðari

$$n_\mathrm{n} = \frac{1}{2}\left[N_\mathrm{D} - N_\mathrm{A} + \sqrt{(N_\mathrm{D} - N_\mathrm{A})^2 + 4n_\mathrm{i}^2}\right]$$

og

$$p_\mathrm{n} = \frac{n_\mathrm{i}^2}{n_\mathrm{n}}$$

p-leiðandi hálfleiðari

$$p_\mathrm{p} = \frac{1}{2}\left[N_\mathrm{A} - N_\mathrm{D} + \sqrt{(N_\mathrm{A} - N_\mathrm{D})^2 + 4n_\mathrm{i}^2}\right]$$

og

$$n_\mathrm{p} = \frac{n_\mathrm{i}^2}{p_\mathrm{p}}$$

$$N_\mathrm{C} = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$N_\mathrm{V} = 2\left(\frac{m_h^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$J = \sigma\mathcal{E}$$

$$\sigma = \frac{nq^2\tau}{m_\mathrm{n}^*} \qquad [\Omega\mathrm{cm}]^{-1}$$

$$\sigma = qn\mu_\mathrm{n}$$

$$\mu_\mathrm{n} = \frac{q\tau}{m_\mathrm{n}^*}$$

$$J = q(n\mu_\mathrm{n} + p\mu_\mathrm{p})\mathcal{E} = \sigma\mathcal{E}$$

$$R = \frac{\rho L}{Wd} = \frac{L}{Wd}\frac{1}{\sigma}$$

3 Viðnám

$$R = \frac{\rho L}{A}$$

$$\sigma = \frac{1}{\rho} = (q\mu_\mathrm{n} n + q\mu_\mathrm{p} p)$$

$$R = \frac{1}{G} = \frac{L}{W}\frac{1}{g}$$