Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2016

Eðlisfræði þéttefnis I

Dæmablað 8

Skilafrestur 1. Nóvember 2016 kl. 15:00

1. **Electron gas in two dimensions** (20)

   We consider a two dimensional gas of electrons of mass m with N electrons confined to an area A so there is a density $n = N/A$ of electrons per unit area.

   (a) Express the Fermi wave vector magnitude k_F and the Fermi energy $\mathcal{E}_\mathrm{F}$ in terms of n

   (b) Express the density of levels $g(\mathcal{E})$

   (c) Write the Sommerfeld expansion for n and conclude as to the relation between the chemical potential μ and the Fermi energy $\mathcal{E}_\mathrm{F}$

   (d) Obtain a relation between μ and $\mathcal{E}_\mathrm{F}$ directly from the relation

   $$n = \int_{-\infty}^{\infty} \mathrm{d}\mathcal{E} g(\mathcal{E}) f(\mathcal{E})$$

   where $f(\mathcal{E})$ is the Fermi-Dirac occupation factor (Hint: proceed with the change of variable $x = \mathrm{e}^{-(\mathcal{E}-\mu)/k_\mathrm{B}T}$). Write your result to the limit $k_\mathrm{B}T \ll \mathcal{E}_\mathrm{F}$.

   (e) Comment on the difference between your answers to question (c) and (d).

2. **Einstein and quantum theory** (10)

   Describe how Einstein used quantum theory to explain the low-temperature behavior of the specific heat in solids. Use more than four sentences in your response.

3. **Thermal motion and resistivity** (10)

   Show that if the random velocity of the electrons were due to thermal motion of a classical electron gas, the electrical resistivity would increase with the temperature as $T^{3/2}$.

4. **Fermi velocity** (10)

   Estimate the ratio of the drift velocity to the Fermi velocity for a 2 mm^2 Cu wire carrying a 20 A current.

5. **Chemical potential of a 1D electron gas** (20)

   (a) Calculate the lowest order non-vanishing correction in $k_B T/E_F$ to the chemical potential $\mu(T)$ for the one-dimensional degenerate electron gas. Compare the sign of the correction to the three dimensional case and explain why it has the sign that it does.

   (b) Using your result from (a), calculate the heat capacity of the one- dimensional degenerate electron gas, corresponding to a crystal in which there is one atom per unit cell of length a and there is one conduction electron per atom. Compare your answer to the phonon contribution at low temperature. Which is bigger ?