Háskóli Íslands
Raunvísindadeild
Eðlisfræði

Haust 2017

Eðlisfræði þéttefnis I

Dæmablað 10

Skilafrestur 16. Nóvember 2017 kl. 15:00

1. **Conductivity of a metal** (10)

   Notið jöfnuna
   $$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = -e\mathbf{E}$$
   fyrir skriðþunga rafeinda til að finna AC leiðni málms. Svar þitt skal vera á forminu $\sigma(\omega, n, e, \tau)$.

   Use the equation
   $$\frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t} + \frac{\mathbf{p}}{\tau} = -e\mathbf{E}$$
   for the electron momentum to find the AC conductivity of a metal. Your answer should be of the form $\sigma(\omega, n, e, \tau)$.

   (Próf desember 2015)

2. **The Kronig-Penney model** (20)

   Consider an electron in 1D in the presence of the periodic potential (Kronig-Penney model)
   $$U(x) = \sum_{m=-\infty}^{\infty} U_0 \Theta(x - ma)\Theta(ma + b - x)$$
   (a) Restrict your attention to a single unit cell, and write down the boundary conditions for the wave function as required by Bloch's theorem.

(b) Solve the Schrödinger equation by constructing ψ(x) from plane waves and imposing suitable boundary conditions at $x = 0, b, a$. The results is a relation between the Bloch index k and the energy.

(c) Take the limit $b \longrightarrow 0$, $U_0 \longrightarrow \infty$ with $U_0 b \longrightarrow W_0 \frac{\hbar^2 a^{-2}}{m}$. Show that the condition for the Bloch index simplifies to

$$\cos(ka) = \frac{W_0}{qa}\sin(qa) + \cos(qa)$$

where q is related to the eigenenergy $\mathcal{E}$ via $q = (2m\mathcal{E}/\hbar^2)^{1/2}$.

(d) Produce plots of the lowest two energy bands $\mathcal{E}_{nk}(n = 0, 1)$ in the limit of part (c) with $a = 1$, $m = 1$, $\hbar = 1$, and $W_0 = 0.5$.

3. **Electronic Impurity State** (20)

Consider the one-dimensional tight binding Hamiltonian given as

$$H_{n,m} = \epsilon_0 \delta_{n,m} - t(\delta_{n+1,m} - \delta_{n-1,m})$$

Now consider the situation where one of the atoms in the chain (atom $n = 0$) is an impurity such that it has an atomic orbital energy which differs by Δ from all the other atomic orbital energies. In this case the Hamiltonian becomes

$$H_{n,m} = \epsilon_0 \delta_{n,m} - t(\delta_{n+1,m} - \delta_{n-1,m}) + \Delta \delta_{n,m} \delta_{n,0}.$$

(a) Using an ansatz

$$\phi_n = A \exp(-qa|n|)$$

with q real, and a the lattice constant, show that there is a localized eigenstate for any negative Δ, and find the eigenstate energy.

(b) Consider instead a continuum one-dimensional Hamiltonian with a delta-function potential

$$H = -\frac{\hbar}{2m^*}\partial_x^2 + (a\Delta)\delta(x)$$

Similarly show that there is a localized eigenstate for any negative Δ and find its energy. Compare your result to that of part (a).

4. **Periodic potential** (10)

Gerum ráð fyrir einvíðu rafeindakerfi sem hlítir veiku lotubundnu mætti

$$U(x) = U_0 \left[\cos^4\left(\frac{\pi x}{a}\right) - \frac{3}{8}\right]$$

Ákvraðið margfeldni punktana við $k = \pm\pi/a$ og $k = \pm 2\pi/a$. Finnið og teiknið tvístrunarsambandið í fyrsta Brillouin svæðinu. Teiknið orkuna í einingunni $\hbar^2\pi^2/2ma^2$, bylgjuvigur í einingunni $1/a$, og gerið ráð fyrir að $U_0 = 0.1$ í þessum einingum.

Consider a one dimensional electron system subject to a weak periodic potential

$$U(x) = U_0 \left[\cos^4\left(\frac{\pi x}{a}\right) - \frac{3}{8}\right]$$

Determine the degeneracy points at $k = \pm\pi/a$ and $k = \pm 2\pi/a$. Find and plot the dispersions of energy bands in the first Brillouin zone. Plot the energies in units of $\hbar^2\pi^2/2ma^2$, the wave numbers in units of $1/a$, and assume that $U_0 = 0.1$ in these units.

(Próf desember 2016)

5. **Fermi level adjustment in silicon** (10)

Kísilsýni við 300 K er íbætt með rafþega íbót af þéttleika $N_\text{A} = 5 \times 10^{16}$ cm^{-3}. Ákvarða íbótarþéttleika rafgjafa íbótar sem bæta verður við þannig að kísillinn verði $n-$leiðandi og Fermi orkustigið sé 0.12 eV neðan við leiðniborðabrún. Virkur ástandsþéttleiki leiðniborða kísils er $N_\text{C} = 2.86 \times 10^{19}$ cm^{-3}.

A silicon sample at 300 K contains an acceptor impurity concentartion of $N_\text{A} = 5 \times 10^{16}$ cm^{-3}. Determine the concentration of donor impurity atoms that must be added so that the silicon is $n-$type and the Fermi level is 0.12 eV below the conduction band edge.

(Próf maí 2016)