Introduction

- In analog design the transistors are not simply switches
- Second-order effects impact their performance
- With each generation these effects become more significant
- The goal is to develop a circuit model for each device by formulating its operation
  - to do this a good understanding of the underlying principles is necessary

Bipolar Junction Transistors

- The bipolar junction transistor (BJT) is a semiconductor device containing three adjoining alternately doped regions
- The central region is known as base and the outer regions emitter and collector
- The emitter is much more doped than the collector

Bipolar Junction Transistors

- pnp bipolar junction transistor
- circuit symbol, voltage and current polarities keep the device in the active region
Bipolar Junction Transistors

- npn bipolar junction transistor
- circuit symbol, voltage and current polarities keep the device in the active region

Bipolar Junction Transistors

- The bipolar junction transistor has four regions of operation
  - The most common region of operation is the **active region**
    - E-B-junctions are forward biased
    - C-B-junctions are reverse biased
    - gives the largest signal gain
    - almost all linear amplifiers operate in the active region
  - The **saturation region** is when
    - E-B-junctions are forward biased
    - C-B-junctions are forward biased

Bipolar Junction Transistors

- The pnp transistor is basically two very closely spaced p-n-junctions
- The base region is $\sim 1 \mu\text{m}$ thick, so the junctions are very closely spaced and interact with each other
- Thus the transistor is capable of current and voltage gain

Bipolar Junction Transistors

- The **cutoff region**
  - E-B-junctions are reverse biased
  - C-B-junctions are reverse biased
  - represents an “off” state for the transistor as a switch
- The **inverted region** is when
  - E-B-junctions are reverse biased
  - C-B-junctions are forward biased
Bipolar Junction Transistors

- The four operating regions of the bipolar junction transistor

- In circuit applications the transistor typically has a common terminal between input and output

Bipolar Junction Transistors

- This gives three amplifier types
  - common base (CB)
  - common emitter (CE)
  - common collector (CC) (emitter follower)

Bipolar Junction Transistors

- Common emitter is the most popular
- The common emitter amplifier has
  - output variables $v_{EC}$ and $i_C$
  - input variables $v_{EB}$ and $i_B$
- if two of the voltages (or currents) are known the third is also known (Kirchhoff’s laws)
Bipolar Junction Transistors

- The current components within the transistor
- The holes injected from the emitter to base are \( I_{Ep} \)

\[
I_E = I_{Ep} + I_{En}
\]

Bipolar Junction Transistors

- The holes injected from the emitter that reach the collector junction are \( I_{Cp} \)
- The current due to thermally generated electrons near the junction are \( I_{Cn} \)

\[
I_C = I_{Cp} + I_{Cn}
\]

\[
I_B = I_E - I_C = I_{B1} + I_{B2} - I_{B3}
\]

Bipolar Junction Transistors

- The base transport factor is defined as the ratio of the hole current diffusing into the collector to the hole current injected at the E-B junction

\[
\alpha_T = \frac{I_{Cp}}{I_{Ep}}
\]

- It is preferred that \( \alpha_T \) is unity, but due to recombination in base \( \alpha_T \) is slightly less than unity
- The emitter injection efficiency

\[
\gamma = \frac{I_{Ep}}{I_E} = \frac{I_{Ep}}{I_{En} + I_{Ep}}
\]

and measures the injected hole current compared to the total emitter current

Bipolar Junction Transistors

- As the emitter is more heavily doped \( \gamma \rightarrow 1 \) and \( I_{En} \rightarrow 0 \)
- The ratio \( I_C/I_E \) in the active region is defined as the dc alpha

\[
\alpha_{dc} = \frac{I_C}{I_E} = \frac{I_{Cp} + I_{Cn}}{I_{Ep} + I_{En}}
\]

- If the E-B junction is forward biased \( I_{Cp} \gg I_{Cn} \) and

\[
\alpha_{dc} = \frac{I_{Cp}}{I_{Ep} + I_{En}}
\]

which can be written as

\[
\alpha_{dc} = \frac{I_{Cp}}{I_{Ep}} \left[ \frac{1}{1 + I_{En}/I_{Ep}} \right] = \alpha_T \left[ \frac{I_{Ep}}{I_{Ep} + I_{En}} \right]
\]

or

\[
\alpha_{dc} = \gamma \alpha_T
\]
Bipolar Junction Transistors

- Beta is defined as
  \[ \beta_{dc} = \frac{I_C}{I_B} = \frac{I_C}{I_E - I_C} \]
  or
  \[ \beta_{dc} = \frac{I_C/I_E}{1 - I_C/I_E} = \frac{\alpha_{dc}}{1 - \alpha_{dc}} \]

- Note that as \( \alpha_{dc} \to 1 \) then \( \beta_{dc} \to \infty \)

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Bipolar Junction Transistors

- The emitter current crossing the E-B depletion region is evaluated as two minority carrier diffusion currents
  \[ I_E = I_{Ep}(0) + I_{En}(0') \]
  or
  \[ I_E = -qA_D B \frac{d\Delta p_B}{dx} \bigg|_{x=0} - qA_D E \frac{d\Delta n_E}{dx'} \bigg|_{x'=0} \]

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Bipolar Junction Transistors

- For holes in the base region
  \[ D_B \frac{d^2\Delta p_B(x)}{dx^2} = \frac{\Delta p_B(x)}{\tau_B} \]
  or
  \[ \frac{d^2\Delta p_B(x)}{dx^2} = \frac{\Delta p_B(x)}{L_B^2} \]
  where \( L_B = \sqrt{D_B \tau_B} \) is the minority carrier diffusion length

- The solution is
  \[ \Delta p_B(x) = C_1 \exp(x/L_B) + C_2 \exp(-x/L_B) \]

- Similar equations can be found for \( \Delta n_E(x') \) and \( \Delta n_C(x') \)
Bipolar Junction Transistors

- The junction voltage $V_{EB}$ and $V_{CE}$ determine the minority carrier concentrations at the edges of the depletion region
  - give the boundary conditions

- Thus the slope of $\Delta p_B(x)$ is
  \[ \frac{\Delta p_B(0) - \Delta p_B(W)}{W} \]

Bipolar Junction Transistors

- If there is no recombination of holes in the base $\tau_B = \infty$ then
  \[ \frac{d^2 \Delta p_B(x)}{dx^2} = 0 \]
  which has solution of the form
  \[ \Delta p_B(x) = C_1 x + C_2 \]
- The two boundary conditions yield
  \[ \Delta p_B(0) = p_{B0}(\exp(qV_{EB}/kT) - 1) = C_1 \times 0 + C_2 \]
  \[ \Delta p_B(W) = p_{B0}(\exp(qV_{CB}/kT) - 1) = C_1 \times W + C_2 \]
  \[ \Delta p_B(x) = \Delta p_B(0) - \left[ \frac{\Delta p_B(0) - \Delta p_B(W)}{W} \right] x \]

Bipolar Junction Transistors

- Similarly
  \[ \Delta n_E(x'') = C_1 \exp(-x''/L_E) + C_2 \exp(x''/L_E) \]
  and with the boundary conditions
  \[ \Delta n_E(x'') = n_{E0}(\exp(qV_{EB}/kT) - 1) \exp(-x''/L_E) \]
- Thus the electron current is
  \[ I_{En}(x'') = -qAD_E \frac{d\Delta n_E(x'')}{dx} \bigg|_{x''=0} = \frac{qAD_E}{L_E} n_{E0}(\exp(qV_{EB}/kT) - 1) \]
Bipolar Junction Transistors

- Similarly the hole current is
  \[ I_{Ep}(0) = -qAD_B \frac{d\Delta p_B(x)}{dx} \bigg|_{x=0} = qAD_E \frac{\Delta p_B(0) - \Delta p_B(W)}{W} \]
  which gives
  \[ I_{Ep}(0) = \frac{qAD_B}{W} \rho_B \left[ \exp(qV_{EB}/kT) - 1 \right] - \left( \exp(qV_{CB}/kT) - 1 \right) \]

- The total emitter current is the sum of the hole and electron currents
  \[ I_E = qA \left[ \frac{D_{EN_E0}}{L_E} + \frac{D_{BPB0}}{W} \right] \left( \exp(qV_{EB}/kT) - 1 \right) \]
  \[ -qA \frac{D_{BPB0}}{W} \left( \exp(qV_{CB}/kT) - 1 \right) \]

Bipolar Junction Transistors – Ebers – Moll

- The currents are now given different names
  \[ I_E = I_{F0} \left( \exp(qV_{EB}/kT) - 1 \right) \]
  \[ = qA \left[ \frac{D_{EN_E0}}{L_E} + \frac{D_{BPB0}}{W} \right] I_{F0} \times \left( \exp(qV_{EB}/kT) - 1 \right) \]
  and
  \[ \alpha_R I_R = \alpha_R I_{R0} \left( \exp(qV_{CB}/kT) - 1 \right) = -qA \frac{D_{BPB0}}{W} \left( \exp(qV_{CB}/kT) - 1 \right) \]

Bipolar Junction Transistors

- The total collector current is the sum of the hole and electron currents
  \[ I_C = -qA \left[ \frac{D_{CN_C0}}{L_C} + \frac{D_{BPB0}}{W} \right] \left( \exp(qV_{CB}/kT) - 1 \right) \]
  \[ + qA \frac{D_{BPB0}}{W} \left( \exp(qV_{EB}/kT) - 1 \right) \]

- The base current is then
  \[ I_B = -qA \frac{D_{EN_E0}}{L_E} \left( \exp(qV_{EB}/kT) - 1 \right) + qA \frac{D_{CN_C0}}{L_C} \left( \exp(qV_{CB}/kT) - 1 \right) \]

Bipolar Junction Transistors – Ebers – Moll

- The emitter current can be written as
  \[ I_E = I_F - \alpha_R I_R \]
  \[ = qA \left[ \frac{D_{EN_E0}}{L_E} + \frac{D_{BPB0}}{W} \right] I_{F0} \times \left( \exp(qV_{EB}/kT) - 1 \right) \]
  and
  \[ \alpha_F I_F = \alpha_F I_{F0} \left( \exp(qV_{EB}/kT) - 1 \right) = -qA \frac{D_{BPB0}}{W} \left( \exp(qV_{EB}/kT) - 1 \right) \]
Bipolar Junction Transistors – Ebers – Moll

• The emitter current can be written as
  \[ I_E = I_F - \alpha_R I_R \]

• The collector current is then written as
  \[ I_C = \alpha_F I_F - I_R \]

• With
  \[ I_R = I_E - I_C = (1 - \alpha_F)I_F + (1 - \alpha_R)I_R \]

• These equations are known as the **Ebers - Moll equations** for an ideal pnp bipolar junction transistor

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Bipolar Junction Transistors – Ebers – Moll

• Note that
  \[ \alpha_F I_{F0} = \alpha_R I_{R0} = I_S \]

• If
  \[ \beta_F = \frac{\alpha_F}{1 - \alpha_F} \]

and

\[ \beta_R = \frac{\alpha_R}{1 - \alpha_R} \]

then only three numbers are necessary for the Ebers-Moll equations to be completely specified, \( \beta_F, \beta_R, \) and \( I_S \) (or \( \alpha_F, \alpha_R, \) and \( I_S \)) and all other parameters can be calculated

• The direct connection between the doping densities, base width, lifetime, etc. and the Ebers - Moll equations makes them partculary useful in integrated circuit analysis

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Bipolar Junction Transistors – Ebers – Moll

• The **Ebers - Moll circuit** for an ideal pnp bipolar junction transistor

• The complete set of input-output \( I - V \) characteristics can be calculated from these equations

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BJT - Small-signal models

• Analog circuits are often operated with signal levels that are small compared to the bias voltages and currents

• Small signal models are then derived to calculate the circuit gain and terminal impedances

• We recall
  \[ I_C = I_S \exp \frac{V_{BE}}{V_T} \]

• The transconductance is defined as
  \[ g_m = \frac{dI_C}{dV_{BE}} = \frac{dI_S \exp(V_{BE}/V_T)}{dV_{BE}} = \frac{I_S}{V_T} \exp \frac{V_{BE}}{V_T} = \frac{I_C}{V_T} \]

• Note that \( g_m = 38 \) mS for \( I_C = 1 \) mA at 25 °C.
**BJT - Small-signal models**

- A change in the base-emitter voltage $\Delta V_{BE}$ causes a change in the minority carrier charge in the base $\Delta Q_e$.

**BJT - Small-signal models**

- The minority carriers are supplied by the base lead so the device has input capacitance (npn)

$$C_b = \frac{\Delta Q_h}{\Delta V_{BE}} = \tau_F g_m = \tau_F \frac{I_C}{V_T}$$

where

$$\tau_F = \frac{W_B^2}{2D_n}$$

is the base transit time in the forward direction.

- Typical values are
  - $\tau_F = 50 - 500$ ps for npn
  - $\tau_F = 1 - 40$ ps for pnp

**BJT - Small-signal models**

- In the forward active region the base current is related to the collector current

$$I_B = \frac{I_C}{\beta_F}$$

and

$$\Delta I_B = \frac{d}{dI_C} \left( \frac{I_C}{\beta_F} \right) \Delta I_C$$

or

$$\beta_0 = \frac{\Delta I_C}{\Delta I_B} = \left[ \frac{d}{dI_C} \left( \frac{I_C}{\beta_F} \right) \right]^{-1}$$

where $\beta_0$ is the small signal current gain of the transistor.

**BJT - Small-signal models**

- Typical values of $\beta_0$ are close to those of $\beta_F$.
  - If $\beta_F$ is constant then $\beta_F = \beta_0$.
  - A single value of $\beta$ is often assumed for a transistor and then used in both ac and dc calculations.
**BJT - Small-signal models**

- Small-signal input impedance of the device
  
  \[ r_x = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{\Delta V_{BE}}{\Delta I_C} \beta_0 = \frac{\beta_0}{g_m} \]

- The small-signal input shunt resistance for a bipolar transistor depends on the current gain and is inversly proportional to \( I_C \)

- Small changes \( \Delta V_{CE} \) in \( V_{CE} \) lead to changes \( \Delta I_C \) in \( I_C \)
  
  \[ \Delta I_C = \frac{\partial I_C}{\partial V_{CE}} \Delta V_{CE} \]
  
  or
  
  \[ \frac{\Delta V_{CE}}{I_C} = \frac{V_A}{I_C} = r_0 \]

  where \( V_A \) is the **Early voltage**

- Note that \( r_0 \) is inversly proportional to \( I_C \) and thus \( r_0 \) can be related to \( g_m \)
  
  \[ r_0 = \frac{1}{\eta g_m} \]

  where
  
  \[ \eta = \frac{kT}{qV_A} \]

  So if \( V_A = 100 \text{ V} \) then \( \eta = 2.6 \times 10^{-4} \) at 25°C

**BJT - Small-signal models**

- \( r_0 \) is the small-signal output resistance of the transistor
  
  \[ r_0 = \frac{1}{g_m} \frac{V_A}{V_T} \]

- The large-signal characteristics of the transistor when the Early effect is included is
  
  \[ I_C = I_S \exp\left( \frac{V_{BE}}{V_T} \right) \left( 1 + \frac{V_{CE}}{V_A} \right) \]

- Combination of the above small-signal elements yields the small-signal model often referred to as the **hybrid-π model**

- This is the basic model that arises directly from essential processes in the device
BJT - Small-signal models

• Technological limitations in the fabrication of a transistor give rise to a number of parasitic elements that have to be taken into account as well.

BJT - Small-signal models

• An increase $\Delta V_{CE}$ in $V_{CE}$ leads to a decrease $\Delta I_B$ in $I_B$ modelled by:

$$r_\mu = \frac{\Delta V_{CE}}{\Delta I_{B1}} = \frac{\Delta V_{CE}}{\Delta I_C} \cdot \frac{\Delta I_C}{\Delta I_{B1}} = r_0 \frac{\Delta I_C}{\Delta I_{B1}}$$

• Typically $I_{B1} < 0.1 I_B$ so:

$$r_\mu \approx 2\beta_0 r_0 - 5\beta_0 r_0$$

• All pn junctions have voltage dependent capacitance:
  - $C_{je}$ is for the base-emitter junction
  - $C_{\mu}$ is for the base-collector junction
  - $C_{cs}$ is for the collector-substrate junction

BJT - Small-signal models

• The base-collector junction capacitance

$$C_{\mu} = \frac{C_{\mu0}}{(1 - \frac{V}{V_0})^n}$$

where $V$ is forward bias and $n \sim 0.2 - 0.5$

• Furthermore there are resistive parasitics:
  - $r_b$ is series resistance in base
  - $r_c$ is series resistance in collector
  - $r_{ex}$ is series resistance in emitter

• The capacitance $C_\pi$ contains the base-charging capacitance $C_b$ and the emitter-base depletion layer capacitance $C_{je}$

$$C_\pi = C_b + C_{je}$$

BJT - Frequency Response

• The addition of the resistive and capacitive parasitics to the basic small-signal circuit gives the complete small-signal equivalent circuit:

$$\Rightarrow \text{Example 4.1}$$
BJT - Frequency Response

- The parasitic circuit elements influence the high-frequency gain of the transistor.
- The capability of a transistor to handle high frequency is specified by the frequency where the magnitude of the shor-circuit common-emitter gain falls to unity.
- This frequency is the transition frequency $f_T$.
- To determine $f_T$ we neglect $r_{ex}$ and $r_{\mu}$ and redraw the complete small signal equivalent circuit.
- If $r_c$ is assumed small then $r_o$ and $C_{es}$ have no influence.

Then the small signal voltage $v_1$ is:

$$v_1 = \frac{r_\pi}{1 + r_\pi(C_\pi + C_\mu)^s}i_i$$

If the current fed forward through $C_\mu$ is neglected we can write:

$$i_o \approx g_m v_1$$

and thus:

$$i_o \approx \frac{g_m r_\pi}{1 + r_\pi(C_\pi + C_\mu)^s}i_i$$

and the high frequency current gain:

$$\frac{i_o(j\omega)}{i_i} \approx \frac{\beta_0}{1 + \frac{\beta_0(C_\pi + C_\mu)}{g_m}j\omega} = \beta(j\omega)$$

At high frequencies the imaginary part of the denominator dominates and the equation simplifies to:

$$\beta(j\omega) \approx \frac{g_m}{j\omega(C_\pi + C_\mu)}$$

When the gain is unity $|\beta(j\omega)| = 1$ we find:

$$\omega = \frac{g_m}{C_\pi + C_\mu}$$

and

$$f_T = \frac{1}{2\pi} \frac{g_m}{C_\pi + C_\mu}$$
The frequency response of a bipolar junction transistor $|\beta(j\omega)|$

- The frequency $\omega_B$ is defined as the frequency where $|\beta(j\omega)| = \frac{\beta_0}{\sqrt{2}}$ or has fallen 3 dB down from the low frequency value
- That is
  \[ \omega_B = \frac{1}{\beta_0} \frac{g_m}{C_{\pi} + C_\mu} \]

A time constant $\tau_T$ is associated with $\omega_T$

\[ \tau_T = \frac{1}{\omega_T} \]

or

\[ \tau_T = \frac{C_{\pi} + C_\mu}{g_m} = \frac{C_{je} + C_b + C_\mu}{g_m} = \tau_F + \frac{C_{je} + C_\mu}{g_m} \]

$\implies$ Example 4.2

Further reading

This discussion is based on sections 1.1, 1.4, 2.1 - 2.4 and 2.6 in Neudeck (1983). It is worth looking at the original paper by Ebers and Moll (1954). A detailed discussion on the small-signal models and frequency response is found in sections 1.3 and 1.4 of Gray et al. (2001).

References

